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Baryonic response of dense holographic QCD

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Abstract: The response function of a homogeneous and dense hadronic system to a time-dependent (baryon) vector potential is discussed for holographic dense QCD (D4/D8 embedding) both in the confined and deconfined phases. Confined holographic QCD is an uncompressible and static baryonic insulator at large N_c and large λ, with a gapped vector spectrum and a massless pion. Deconfined holographic QCD is a diffusive conductor with restored chiral symmetry and a gapped transverse baryonic current. Similarly, dense D3/D7 is diffusive for any non-zero temperature at large N_c and large λ. At zero temperature dense D3/D7 exhibits a baryonic longitudinal visco-elastic mode with a first sound speed $1/\sqrt{3}$ and a small width due to a shear viscosity to baryon ratio $\eta/n_B = \hbar/4$. This mode is turned diffusive by arbitrarily small temperatures, a hallmark of holography.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence.

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1. Introduction

Solids respond to external stress elastically through their bulk and shear modulii K and μ respectively, with almost zero dissipation. Liquids on the other hand, follow the lore of hydrodynamics with bulk and shear viscosities ξ and η accounting for dissipation. In contrast to the solid, the shear modulus vanishes in the liquid. The bulk modulus does not.

This remarkable difference between solid and liquid disappears when the stress is time-dependent. Indeed, for a stress of finite frequency ω a liquid has a non-zero shear modulus much like the solid. In the long-wavelength limit, the dual description of a solid or a liquid follows from the visco-elastic equations with complex and frequency dependent elastic constants as we detail below. In this paper we will explore some of these ideas in the context of the AdS/CFT correspondence by analyzing the baryonic response functions at finite density for both D4/D8 and D3/D7 embeddings.

The AdS/CFT approach [1] provides a useful framework for discussing large N_c gauge theories at strong coupling $\lambda = g^2 N_c$. The model suggested by Sakai and Sugimoto (SS) [2] offers a specific holographic realization of hQCD that includes N_f flavors and is chiral. For $N_f \ll N_c$, chiral QCD is obtained as a gravity dual to N_f D8-$\overline{\text{D8}}$ branes embedded into a $D4$ background in 10 dimensions where supersymmetry is broken by the Kaluza-Klein (KK) mechanism. The SS model yields a holographic description of hadrons in the vacuum [2 − 7], at finite temperature [8] and finite baryon density [9 − 12].

Hot and dense hadronic matter in QCD is difficult to track from first principles in current lattice simulations owing to the sign problem. In large N_c QCD baryons are solitons and a dense matter description using Skyrme's chiral model [13] was originally suggested by Skyrme and others [14]. At large N_c and low density matter consisting of solitons crystallizes as the ratio of potential to kinetic energy $\Gamma = V/K \approx N_c^2/p_F^2 \gg 1$ is much larger than 1. The crystal melts at sufficiently high density with $\Gamma \approx N_c^2/p_F^2 \approx 1$, or sufficiently high temperature with $\Gamma \approx N_c/T \approx 1$. QCD matter at large N_c was recently revisited in [15].

The many-soliton problem can be simplified in the crystal limit by first considering all solitons to be the same and second by reducing the crystal to a single cell with boundary conditions much like the Wigner-Seitz approximation in the theory of solids. A natural way to describe the crystal topology is through T^3 with periodic boundary conditions. A much simpler and analytically tractable approximation consists of treating each Wigner-Seitz cell as S^3 with no boundary condition involved. The result is dense Skyrmion matter on S^3 [16].

At low baryonic densities holographic QCD is a crystal of instantons with the Wigner-Seitz cell *approximated* by S^3. The pertinent instanton is defined on $S^3 \times R$ [11]. At moderate densities chiral symmetry is restored on the average with an $n_B^{5/3}$ equation of state [11]. This homogenous (on the average) liquid-like phase is strongly coupled and not emmenable to standard Fermi liquid analysis.

In this paper, we would like to follow up on the transport properties in the homogeneous phase originally discussed in [10] using D4/D8 to contrast them with some recent studies in [17] using D3/D7. In section 2, we recall the bulk characteristics of the homo-

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geneous phase in D4/D8 and suggest that it may be identified with a strongly coupled holographic liquid prior to the restoration of chiral symmetry. In section 3, we derive the general formulae for the holographic currents induced by an external baryonic field in the linear response approximation for both D4/D8 and D3/D7. In section 4 we show that the transverse baryonic current for cold D4/D8 is saturated by medium modified vector mesons in leading N_c in agreement with [10]. The bulk static conductivity is zero. Large N_c D4/D8 is an insulator. In section 5, we develop the quasi-normal mode approach for hot and dense D4/D8 and D3/D7, both of which are conductors at large N_c. For completeness we also discuss cold D3/D7 in light of a recent result [17]. In section 6, we suggest a unified visco-elastic framework for interpreting gapless excitations in dense media in both the elastic (collisionless) and hydrodynamic (collision) regimes. We argue that cold D3/D7 exhibits such a mode at large N_c with zero bulk viscosity and finite shear viscosity. In section 7, we suggest that the leading $1/N_c$ correction to the baryonic currents in cold D4/D8 can be extracted from an effective baryonic theory using the Random Phase Approximation. Our conclusions and prospects are in section 8. A number of points pertaining to transport in dense holographic media are discussed in the appendices.

2. Homogeneous dense matter

First we briefly review the bulk property of the homogeneous phase in the SS model. More details can be found in [10]. We consider Sakai-Sugimoto's original embedding [2, 3], where the D8-branes configuration in the τ coordinate is constant and not affected by the existence of a background $\mathrm{U}(1)_V$ field $\mathbb{A}_0$. This corresponds to $\tau = \delta\tau/4$, the maximal asymptotic separation between D8 and $\overline{\mathrm{D8}}$ branes. The DBI action of D8 branes with $\mathbb{A}_0$ is written as[1]

$$S_{\mathrm{DBI}} = -a \int d^4x \int dZ\, K^{2/3} \sqrt{1 - bK^{1/3}(\partial_Z \mathbb{A}_0)^2}\,, \tag{2.1}$$

where

$$a \equiv \frac{N_c N_f \lambda^3 M_{\mathrm{KK}}^4}{3^9 \pi^5}\,, \qquad b \equiv \frac{3^6 \pi^2}{4\lambda^2 M_{\mathrm{KK}}^2}\,, \qquad K = 1 + Z^2\,. \tag{2.2}$$

M_{KK} is the Kaluza-Klein mass and λ is t'Hooft coupling. Now we introduce the baryon source coupled to $\mathbb{A}_0$ through the Chern-Simons term. We assume that baryons are uniformly distributed over $\mathbb{R}^3$ space whose volume is V. For large λ, the instanton size is $1/\sqrt{\lambda}$ [4, 5]. It can be treated as a static delta function source at large N_c. For a uniform baryon distribution, the source is

$$S_{\mathrm{source}} = N_c n_B \int d^4x \int dZ\, \delta(Z) \mathbb{A}_0(Z)\,. \tag{2.3}$$

By varing the total action $S_{\mathrm{DBI}} + S_{\mathrm{source}}$ we get the classical solution $\mathbb{A}_0$:

$$\mathbb{A}_0(Z; n_q) = \int_0^Z dZ \frac{n_q/2}{\sqrt{(ab)^2 K^2 + bK^{1/3} n_q^2/4}}\,, \tag{2.4}$$

[1]The integral is extended to $(-\infty, \infty)$ to take into account $\overline{\mathrm{D8}}$ branes as well as D8 branes.

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which defines the baryon *quark* chemical potential μ_q as

$$\mu_q(n_q) \equiv \lim_{|Z|\to\infty} \mathbb{A}_0(Z; n_q) \ . \tag{2.5}$$

This relation also defines μ_q as a function of n_q and vice versa. The baryon chemical potential μ_B is

$$\mu_B = m_B + N_c \mu_q \,, \tag{2.6}$$

where m_B is the baryon rest mass.

The interaction energy density $\epsilon_{\rm int}$, pressure P, grand potential Ω, and the baryon chemical potential μ_B have been computed in [10],

$$\epsilon_{\rm int} \equiv \frac{\Delta E}{V} = a \int_{-\infty}^{\infty} dZ K^{2/3} \left[\sqrt{1 + \frac{(N_c n_B)^2}{4a^2 b} K^{-5/3}} - 1 \right] \,, \tag{2.7}$$

$$P = -\frac{\Omega}{V} = a \int_{-\infty}^{\infty} dZ\, K^{2/3} \left[1 - \frac{1}{\sqrt{1 + \frac{(N_c n_B)^2}{4a^2 b} K^{-5/3}}} \right] \,, \tag{2.8}$$

$$\mu_B = m_B + N_c \int_{-\infty}^{\infty} dZ \frac{N_c n_B/4}{\sqrt{(ab)^2 K^2 + b K^{1/3} (N_c n_B/2)^2}} \,, \tag{2.9}$$

where V is the volume and Ω is understood as a function of μ_B through (2.5) and (2.6). At low densities they translate to

$$\epsilon_{\rm int} \sim \frac{27\pi^3}{2} \frac{N_c}{N_f \lambda} \frac{1}{M_{\rm KK}^2} n_B^2 \,, \tag{2.10}$$

$$P = -\frac{\Omega}{V} \sim \frac{27\pi^3}{2} \frac{N_c}{N_f \lambda} \frac{1}{M_{\rm KK}^2} n_B^2 \,, \tag{2.11}$$

$$\mu_B \sim m_B + 27\pi^3 \frac{N_c}{N_f \lambda} \frac{1}{M_{\rm KK}^2} n_B \ . \tag{2.12}$$

The baryonic contributions appear through the combination $N_c/\lambda N_f$. The large N_c and large λ limit are not compatible in the homogeneous phase. Compatibility with solitonic physics suggests that the large N_c limit be taken first followed by the large λ limit, which is also consistent with holography. This will be assumed throughout, unless specified otherwise. The homogeneous phase described by (2.8) breaks spontaneously chiral symmetry with density dependent vacuum-like modes [10]. In figure 1 we sketch the various phases of dense holographic matter at zero temperature. The low density part is inhomogeneous (solid) with spontaneously broken chiral symmetry, while the high density phase is homogeneous (gas) with restored chiral symmetry. Intermediate between the two is a possible liquid phase. Here we suggest that (2.8) may capture some aspects of the liquid phase still in the spontaneously broken phase using holography. The solid phase binds with an energy density $\epsilon_{\rm int} \approx a_M\, n_B$ where a_M is the Madelung constant for the pertinent crystallization provided that the baryons are semiclassically quantized to account for the pion-interaction through the mesonic cloud [6]. The gas phase is homogeneous with $\epsilon_{\rm int} \approx n_B^{5/3}$ and restored chiral symmetry [11]

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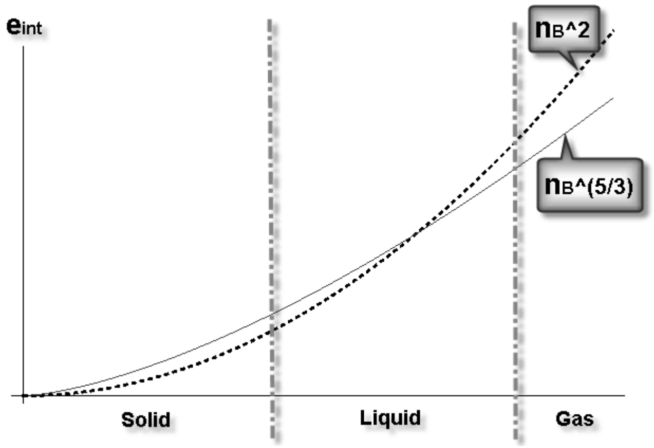

Figure 1: Sketch of the phases of cold D4/D8.

2.1 Compressibility

Holographic QCD at large N_c and large λ is umcompressible. Indeed, under small scalar or longitudinal vector stress the baryonic density n_B is expected to change locally to $n_B+\delta n_B$ so that the constitutive equations read

$$M n_B \dot{\vec{v}} = -\vec{\nabla} p\,, \tag{2.13}$$

$$\partial_t \delta n_B + n_B \vec{\nabla}\cdot\vec{v} = 0\,, \tag{2.14}$$

by the Newtonian equation of motion (2.13) and baryon current conservation (2.14). The baryonic charges move with an acceleration $(\partial P/\partial n_B)/m_B \approx 1/\lambda$ which is suppressed at large λ since $m_B = 8\pi\lambda N_c$ [5]. Another way to say this is to note that (2.14) implies $(\partial_t^2 - c_1^2\nabla^2)\delta n_B = 0$, with the speed of the first or thermodynamic sound $c_1 = \sqrt{\partial P/\partial n_B/m_B}$. For the confined D4/D8 configuration

$$c_1 = \left(\frac{27\pi}{8}\frac{n_B}{\lambda^2 N_f}\int dZ \frac{1}{K}\frac{1}{\sqrt{1+\frac{N_c^2 n_B^2}{4a^2 b}K^{-5/3}}^{\,3}}\right)^{1/2}\,, \tag{2.15}$$

after using (2.8). The bulk modulus is $\mathbf{K} = n_B \partial P/\partial n_B \approx n_B^2(N_c/\lambda)$, with the compressibility $\chi = 1/\mathbf{K} \approx (\lambda/N_c)/n_B^2$. Holographic QCD is uncompressible at large N_c.

3. Holographic baryonic currents

Baryon transport in confined D4/D8 occurs explicitly through $1/N_c$ effects. Contributions to the baryonic current to order N_c^0 are shown in figure 2. They follow from direct (a) or vector meson (b) such as the ω meson. All density effects in holography are suppressed at large N_c and large λ. To illustrate these points, we streamline the dense analysis given in [10] using general notations to extend the results to finite temperature and also other brane embeddings. The induced metric on the D8 branes for both low (KK) and high

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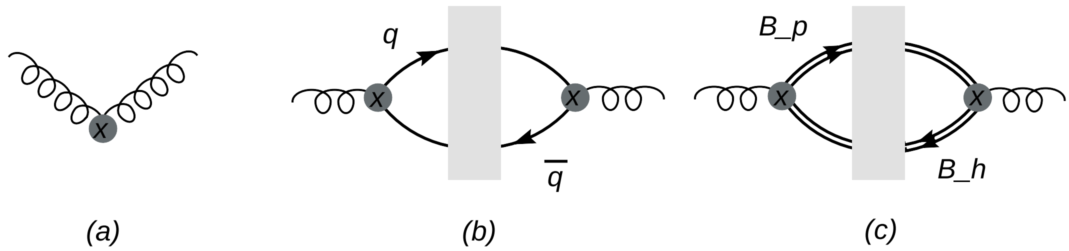

Figure 2: Typical contributions to the baryonic response in D4/D8. (a) Direct N_c^0 (b) Vector mesons N_c^0 and (c) Fermi (baryon) contributions N_c^{-1}

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temperatures(BH) can be written as

$$ds^2_{\rm D8} \equiv g_{tt}dt^2 + g_{xx}\delta_{ij}dx^i dx^j + g_{UU}\, dU^2 + g_{SS}d\Omega_4^2 \tag{3.1}$$

$$\equiv \alpha\left(\frac{U}{R}\right)^{3/2} dt^2 + \left(\frac{U}{R}\right)^{3/2}\delta_{ij}dx^i dx^j + \left(\frac{R}{U}\right)^{3/2}\gamma\, dU^2 + \left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2\,, \tag{3.2}$$

where for the KK background

$$\alpha \;\rightarrow\; -1\;, \quad \gamma \;\rightarrow\; \frac{1}{f(U)} + \left(\frac{\partial x^4}{\partial U}\right)^2\left(\frac{U}{R}\right)^3 f(U)\,, \quad f(U) \;\rightarrow\; 1-\left(\frac{U_{\rm KK}}{U}\right)^3\,, \tag{3.3}$$

and for the BH background

$$\alpha \rightarrow -f(U)\;, \quad \gamma \rightarrow \frac{1}{f(U)} + \left(\frac{\partial x^4}{\partial U}\right)^2\left(\frac{U}{R}\right)^3\,, \quad f(U) \;\rightarrow\; 1-\left(\frac{U_T}{U}\right)^3\,. \tag{3.4}$$

The embedding information is only encoded in γ and thereby g_{UU}.

With the induced metric (3.1) and the pertinent guage fields, the general DBI action follows as

$$S_{\rm DBI} = -\mathcal{N}\ {\rm tr}\int d^4x dU\ e^{-\phi} g_{SS}^2\Big[-g_{00}g_{xx}^3 g_{UU} - g_{xx}^3 F_{0U}F_{0U} - g_{xx}^2 g_{UU}\sum_i F_{0i}F_{0i}$$
$$-g_{00}g_{xx}^2\sum_i F_{iU}F_{iU} - g_{00}g_{xx}g_{UU}\sum_{i>j}F_{ij}F_{ij} - g_{xx}\sum_{i>j}F_{ij}F_{ij}F_{U0}F_{U0} + \cdots\Big]^{1/2} \tag{3.5}$$

where $e^{-\phi} = g_s(U/R)^{3/4}$ and $\mathcal{N} \equiv T_8\Omega_4$. The D_8 brane tension is T_8 and Ω_4 is the volume of a unit S^4.[2] The F^3 and F^5 terms cancel by symmetry. Among the F^4 terms we only retained the relevant term for our discussion below. If we consider the fluctuation $(A_\alpha(x^\alpha))$ around the classical configuration $\mathbb{A}_0$ (2.4), which is due to homogeneous matter at $Z = 0$, the action can be expanded as

$$\frac{R^{2/3}\mathcal{N}}{2g_s}(2\pi\alpha')^2{\rm tr}\int dU U^{5/2}\frac{1}{\sqrt{-\alpha\gamma}}\Big[\frac{2\alpha\gamma\Delta^{-1}}{(2\pi\alpha')^2} + 2\Delta(\partial_U\mathbb{A}_0)F_{U0} + \Delta^3 F_{U0}F_{U0}$$
$$+\Delta\alpha\sum_i F_{iU}F_{iU} + \Delta\gamma\left(\frac{R}{U}\right)^3\sum_i F_{0i}F_{0i} + \Delta^{-1}\alpha\gamma\left(\frac{R}{U}\right)^3\sum_{i>j}F_{ij}F_{ij}\Big]\,, \tag{3.6}$$

[2] We absorb $2\pi\alpha'$ into the gauge field for notational convenience. It will be recalled in the final physical quantities.

	N	k_1	k_2	k_3	$2\pi\alpha'\mathbb{A}_0'$	Δ
D4/D8$_{\rm con}$	$\kappa \equiv \frac{\lambda N_c}{216\pi^3}$	K	$K^{-4/3}M_{\rm KK}^{-2}$	-1	$\frac{d}{\sqrt{b}\sqrt{K^2+K^{1/3}d^2}}$	$\sqrt{1+d^2K^{-5/3}}$
D4/D8$_{\rm dec}$	$\frac{\lambda N_c T^3}{54\pi}$	$\frac{K^{3/2}}{\sqrt{K-1}}$	$K^{-4/3}(2\pi T)^{-2}$	$-\frac{K-1}{K}$	$\frac{d}{\sqrt{K^{5/3}+d^2}}$	$\sqrt{1+d^2K^{-5/3}}$
D3/D7	$\frac{\lambda N_f N_c}{2(2\pi)^4}$	Z^3	$Z^{-4}f^{-1}$	$-f$	$\frac{d}{\sqrt{Z^6+d^2}}$	$\sqrt{1+d^2Z^{-6}}$

Table 1: Parameters of the different embeddings in (3.10). See text.

up to quadratic terms. $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha - i[A_\alpha, A_\beta]$ and

$$\Delta \equiv \frac{1}{\sqrt{1+\frac{(2\pi\alpha')^2}{\alpha\gamma}(\mathbb{A}_0')^2}} \ . \tag{3.7}$$

It is useful to change variable

$$U = U_0(1+Z^2)^{1/3} \,, \tag{3.8}$$

where U_0 is the coordinate of the tip of the D8-$\overline{\rm D8}$ cigar-shaped configuration in the confining background and the position of the horizon U_T in the black hole background. The range of Z is $(0,\infty)$ contrary to U whose range is (U_0,∞). Also this range can be extended to $(-\infty,\infty)$ if we consider $\overline{\rm D8}$ branes $(-\infty,0)$ together with D8 branes $(0,\infty)$ in a natural way. For convenience, we note the following useful relations

$$K \equiv 1+Z^2 \,, \quad U = U_0 K^{1/3} \,, \quad dU = \frac{2U_0}{3}\frac{Z}{K^{2/3}}dZ \,, \quad f = 1-\left(\frac{U_*}{U_0}\right)^3\frac{1}{K} \,, \tag{3.9}$$

where U_* is U_T for the black hole background. For the confining background, from here on and for simplicity, we follow Sakai and Sugimoto [2] and choose $U_* = U_{\rm KK}$. In terms of Z the action reads

$$S = N{\rm tr}\int d^4x dZ k_1\Big[2\Delta\mathbb{A}_0' F_{Z0} + \Delta^3 F_{Z0}F_{Z0} + \Delta k_2 \sum_i F_{0i}F_{0i} + \Delta k_3 \sum_i F_{iZ}F_{iZ} + \Delta^{-1}k_2k_3\sum_{i>j}F_{ij}F_{ij}\Big] \,, \tag{3.10}$$

where we dropped the fluctuation independent part and the parameters $(k_1, k_2, k_3, \Delta, N)$ are different for each of the brane embeddings. They are summarized in table 1. The case $D3/D7$ is separatly discussed below (section 5.3). We note that the dimensionless densities are $d = \frac{3^6\pi^4}{\lambda^2 N_f M_{\rm KK}^3} n_B$ for D4/D8$_{\rm con}$, $d = \frac{3^5\pi^2}{2^5 N_c N_f \lambda T^5 l_s^2} n_B$ for D4/D8$_{\rm dec}$, and $d = \frac{(2\pi)^3}{\sqrt{\lambda}N_f N_c} n_q$ for D3/D7 with $f = 1-\frac{Z_H^4}{Z^4}$. This explicitly shows that the density effects are subleading at large λ and large N_c for fixed N_f.

Now consider an abelian fluctuation in the $A_Z = 0$ gauge

$$A_\mu = a_\mu(x^0, x^3, Z) + \mathcal{V}_\mu(x^0, x^3) \,, \tag{3.11}$$

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where a_μ vanishes at the boundary i.e. $a_\mu(x^0, x^3, \infty) = 0$, so that the boundary field is simply $\mathcal{V}_\mu(x^0, x^3)$. $\mathcal{V}_\mu(x^0, x^3)$ exists for all Z as the background.[3] With the Fourier decomposition

$$a_\mu(Z, x^0, x^3) = \int \frac{d\omega dq}{(2\pi)^2} e^{-i\omega x^0 + iqx^3} a_\mu(Z, \omega, q)\,, \tag{3.12}$$

$$\mathcal{V}_\mu(Z, x^0, x^3) = \int \frac{d\omega dq}{(2\pi)^2} e^{-i\omega x^0 + iqx^3} \mathcal{V}_\mu(\omega, q)\,, \tag{3.13}$$

the quadratic action can be rewritten as[4]

$$S = N \int \frac{d\omega dq}{(2\pi)^2} dZ \Big[a_L \mathcal{D}_L a_L - 2 f_L \mathcal{V}_L a_L - f_L \mathcal{V}_L \mathcal{V}_L - 2 g_L a_L + a_T \mathcal{D}_T a_T - 2 f_T \mathcal{V}_T a_T - f_T \mathcal{V}_T \mathcal{V}_T \Big]\,,$$

where we introduced the gauge invariant variables

$$\begin{aligned} &\text{Longitudinal mode:} \quad a_L \equiv q a_0 + \omega a_3, \qquad \mathcal{V}_L \equiv q \mathcal{V}_0 + \omega \mathcal{V}_3\,, \\ &\text{Transverse mode:} \quad a_T \equiv \omega a_1, \qquad V_T \equiv \omega \mathcal{V}_1\,, \end{aligned} \tag{3.14}$$

with $a_2 = 0$ and used Gauss constraint $\Delta^2 \omega a_0' + k_3 q a_3' = 0$. $a_2 = 0$ is a consistent choice since the transversal equation of motion decouples from the others.

The differential operators $\mathcal{D}_{L/T}$ are defined as

$$\mathcal{D}_L \equiv \partial_Z \frac{-k_1 k_3 \Delta^3}{\Delta^2 \omega^2 + k_3 q^2} \partial_Z + k_1 k_2 \Delta\,, \tag{3.15}$$

$$\mathcal{D}_T \equiv \frac{1}{\omega^2} \Big(\partial_Z k_1 k_3 \Delta \partial_Z - k_1 k_2 (\Delta \omega^2 + \Delta^{-1} k_3 q^2) \Big)\,, \tag{3.16}$$

and the coefficient functions are

$$f_L \equiv -k_1 k_2 \Delta\,, \qquad g_L \equiv \frac{k_3 q \mathbb{A}_0'}{\Delta^2 \omega^2 + k_3 q^2}\,, \tag{3.17}$$

$$f_T \equiv \frac{k_1 k_2 (\Delta \omega^2 + \Delta^{-1} k_3 q^2)}{\omega^2}\,. \tag{3.18}$$

The equations of motion is

$$\mathcal{D}_L a_L = f_L \mathcal{V}_L + g_L\,, \qquad \mathcal{D}_T a_T = f_T \mathcal{V}_T. \tag{3.19}$$

With the formal solutions

$$a_L = \mathcal{D}_L^{-1}(f_L \mathcal{V}_L + g_L)\,, \qquad a_T = \mathcal{D}_T^{-1}(f_T \mathcal{V}_T)\,, \tag{3.20}$$

[3]This is equivalent to the usual set up A_μ with the boundary condition $A_\mu(x^0, x^3, \infty) = \mathcal{V}_\mu(x^0, x^3)$. The difference is in the equations of motion. The equation for A_μ is homogeneous but the equation for a_μ is inhomogeneous and sourced by $\mathcal{V}_\mu$ as shown in (3.19).

[4]For simplicity we omitted the argument of the functions. Each quadratic term is a function of (ω, q) (first) and $(-\omega, -q)$ (second). We dropped the surface terms since they vanish on shell when the source is explicitly present.

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the on-shell action reads

$$S = -N \int \frac{d\omega dq}{(2\pi)^2} dZ \Big(\mathcal{V}_L f_L [\mathcal{D}_L^{-1}(f_L \mathcal{V}_L) + \mathcal{V}_L] + \mathcal{V}_L [2 f_L \mathcal{D}_L^{-1} g_L] + g_L \mathcal{D}_L^{-1} g_L \\ + \mathcal{V}_T f_T [\mathcal{D}_T^{-1}(f_T \mathcal{V}_T) + \mathcal{V}_T] \Big) . \tag{3.21}$$

The induced baryonic currents follow to leading order in large N_c and large λ as

$$J_L(\omega, q) = 2N\omega \int dZ f_L [\mathcal{D}_L^{-1}(f_L \mathcal{V}_L + g_L) + \mathcal{V}_L] , \tag{3.22}$$

$$J_T(\omega, q) = 2N\omega \int dZ f_T [\mathcal{D}_T^{-1}(f_T \mathcal{V}_T) + \mathcal{V}_T] , \tag{3.23}$$

where $\mathcal{D}_L^{-1}$ and $\mathcal{D}_T^{-1}$ are understood with the retarded prescription $\omega \to \omega + i0$. f_L, f_T, g_L are all recorded in (3.18). The longitudinal current involves g_L independently of $\mathcal{V}_L$ as g_L is triggered by the *gradient* of the baryonic profile $\mathbb{A}_0'$. This is the analogue of Fick's law (baryonic charge diffusion). The terms involving $\mathcal{V}_{L,T}$ correspond to $\sigma_{L,T}$ the longitudinal and transverse Fourier transforms of the space-time conductivities. The arguments (ω, q) are subsumed.

4. D4/D8: cold

In the confined phase, the operators $\mathcal{D}_{L,T}$ are hermitian modulo the retarde prescription in frequency space. They can be diagonalized using eigenmodes as discussed in [10]. Throughout the prescription $\omega \to \omega + i0$ is subsumed.

4.1 Longitudinal mode

The longitudinal operator $(\mathcal{D}_L)$ is

$$\mathcal{D}_L \equiv \partial_Z \frac{K\Delta^3}{\Delta^2\omega^2 - q^2} \partial_Z + K^{-1/3}\Delta . \tag{4.1}$$

When $q = 0$ or $\omega = 0$ it is easily diagonalized, since

$$\begin{aligned} \mathcal{D}_L(q=0) &= \frac{1}{\omega^2} \partial_Z K\Delta \partial_Z + K^{-1/3}\Delta , \\ \mathcal{D}_L(\omega=0) &= -\frac{1}{q^2} \partial_Z K\Delta^3 \partial_Z + K^{-1/3}\Delta . \end{aligned} \tag{4.2}$$

The Green's function $(\mathcal{D}_L^{-1})$ may be expanded in terms of the complete set of eigenvalues that diagonalize

$$\begin{aligned} \mathcal{D}_L(q=0) f &= \left(\frac{K^{-1/3}\Delta}{\omega^2} \right) \lambda f , \\ \mathcal{D}_L(\omega=0) f &= \left(\frac{K^{-1/3}\Delta}{q^2} \right) \lambda f , \end{aligned} \tag{4.3}$$

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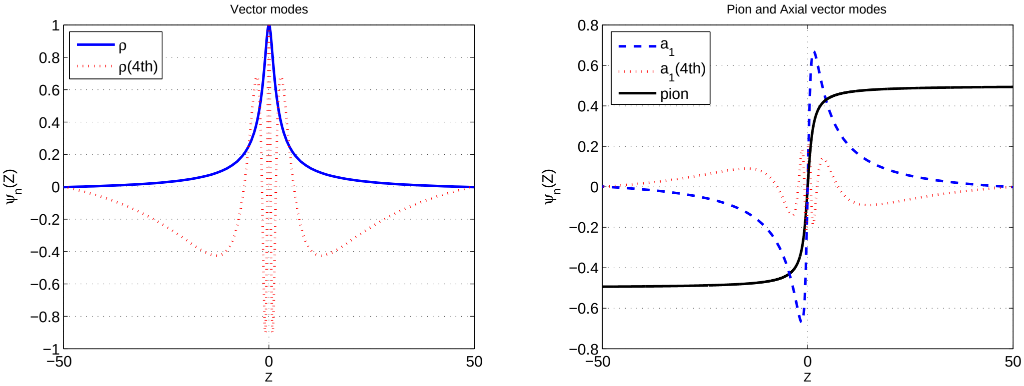

Figure 3: Vector mode functions (Left) and axial-vector and pion mode functions (Right)

where $\frac{K^{-1/3}\Delta}{\omega^2}$ and $\frac{K^{-1/3}\Delta}{q^2}$ are weight factors. Using the complete sets,

$$\begin{aligned}(\partial_Z K\Delta\partial_Z)\,\chi_n &= -(K^{-1/3}\Delta)\,(\lambda_n^{\chi})^2\,\chi_n\,,\\ (\partial_Z K\Delta^3\partial_Z)\,\xi_n &= -(K^{-1/3}\Delta)\,(\lambda_n^{\xi})^2\,\xi_n\,,\end{aligned} \tag{4.4}$$

we have

$$\langle Z|\mathcal{D}_L^{-1}(q=0)|Z'\rangle = \sum_{n\in\mathbb{N}} \frac{\chi_n(Z)\chi_n(Z')}{-\omega^2+(\lambda_n^{\chi})^2} + \frac{\chi_0(Z)\chi_0(Z')}{-\omega^2}\,, \tag{4.5}$$

$$\langle Z|\mathcal{D}_L^{-1}(\omega=0)|Z'\rangle = \sum_{n\in\mathbb{N}} \frac{\xi_n(Z)\xi_n(Z')}{q^2+(\lambda_n^{\xi})^2} + \frac{\xi_0(Z)\xi_0(Z')}{q^2}\,, \tag{4.6}$$

which are the results of [10]. Typical behaviors of χ_n and ξ_n are shown in figure 3.

For small ω, q we may write without loss of generality,

$$\begin{aligned}\langle Z|\mathcal{D}_L^{-1}(\omega\approx q\approx 0)|Z'\rangle \approx &\sum_{n\in 2\mathbb{N}} \frac{\varphi_n(Z)\varphi_n(Z')}{-\omega^2+c_n^2q^2+(\lambda_n^{\chi})^2} + \frac{\varphi_0(Z)\varphi_0(Z')}{-\omega^2+c_\pi^2q^2}\\ &+\sum_{n\in 2\mathbb{N}+1} \frac{\varphi_n(Z)\varphi_n(Z')}{-\omega^2+c_n^2q^2+(\lambda_n^{\chi})^2}\,.\end{aligned} \tag{4.7}$$

The first contribution is from the density dependent axial-vector mode, the second contribution is from the density dependent pion mode (strictly speaking its U(1) partner at large N_c), and the last contribution is from the density dependent vector mode. The denominators are the dispersive modes, while the numerators capture their residues. The even-odd in the labelling of the modes translates into odd-even in the parity of $\varphi_n(Z)$. The baryonic current reads

$$J_L = 2N\omega \int dZ K^{-1/3}\Delta \left[\mathcal{D}_L^{-1}(K^{-1/3}\Delta\mathcal{V}_L + g_L) + \mathcal{V}_L\right]\,. \tag{4.8}$$

The Z-integration picks only the vector or even-modes of (4.5) since $\mathcal{V}_L$ is trivially even. The longitudinal baryonic conductivity in the confined case is

$$\sigma_L = 2N\omega \int dZ K^{-1/3}\Delta \left[1 + \mathcal{D}_L^{-1}K^{-1/3}\Delta\right]\,. \tag{4.9}$$

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The longitudinal mesonic propagator $\mathcal{D}_L^{-1}$ admits the mode decomposition (4.7). From (4.4) it follows that

$$\int dZ f_L \chi_n = \frac{1}{\lambda_n^2}\int dZ\,(\partial_Z K\Delta\partial_Z)\,\chi_n = \frac{1}{\lambda_n^2}\,(K\Delta\partial_Z\,\chi_n)_{-\infty}^{+\infty} = 0$$

is a zero boundary term. The longitudinal baryonic conductivity simplifies

$$\sigma_L = -2N\omega\int dZ f_L\,, \tag{4.10}$$

and so does the longitudinal current. The longitudinal conductivity vanishes at $\omega = 0$. Confined holographic QCD is a static insulator at large N_c and large λ in agreement with our recent analysis [12].

We now note that

$$c_n \equiv \frac{\lambda_n^\chi}{\lambda_n^\xi}\,, \qquad c_\pi \equiv \frac{f_\pi^S}{f_\pi^T} = \sqrt{\frac{\int dZ K^{-1}\Delta^{-3}}{\int dZ K^{-1}\Delta^{-1}}}, \tag{4.11}$$

where f_π^S and f_π^T have been derived in [10]. At high density the pion speed vanishes as $c_\pi \approx 1/n_B$. The propagation of the axial charge stalls in very dense matter. For small momenta q the poles develop at

$$\omega_n \approx \sqrt{(c_n^2 q^2 + \lambda_n^\chi)^2} \approx \lambda_n^\chi + \frac{1}{2}\frac{c_n^2 q^2}{\lambda_n^\chi}\,, \tag{4.12}$$

while for small frequencies ω

$$c_n^2(-(\lambda_n^\xi)^2) + (\lambda_n^\chi)^2 = 0 \;\Rightarrow c_n^2 = \left(\frac{\lambda_n^\chi}{\lambda_n^\xi}\right)^2\,, \tag{4.13}$$

since $q_n^2 = -(\lambda_n^\xi)^2$ from (4.6). In the confined D4/D8 embedding, the vector and axial modes disperse through

$$\omega_n \approx \lambda_n^\chi + \frac{1}{2}\frac{\lambda_n^\chi}{(\lambda_n^\xi)^2}q^2\,, \tag{4.14}$$

where λ_n^χ is the rest mass and $\frac{(\lambda_n^\xi)^2}{\lambda_n^\chi}$ is the kinetic mass (figure 4). To this order, the imaginary parts vanish in holographic QCD [10]. Indeed, vector, axial-vector and pionic modes are expected to be absorbed by excited and/or recoiling baryons which are $1/N_c$ suppressed effects in cold and dense QCD.

4.2 Transverse mode

The transverse operator ($\mathcal{D}_T$) is given by

$$\mathcal{D}_T \equiv -\frac{1}{\omega^2}\left(\partial_Z K\Delta\partial_Z + K^{-1/3}(\Delta\omega^2 - \Delta^{-1}q^2)\right)\,. \tag{4.15}$$

For $q = 0$ it diagonalizes trivially through

$$\mathcal{D}_T(q=0) \equiv -\frac{1}{\omega^2}\partial_Z K\Delta\partial_Z - K^{-1/3}\Delta\,, \tag{4.16}$$

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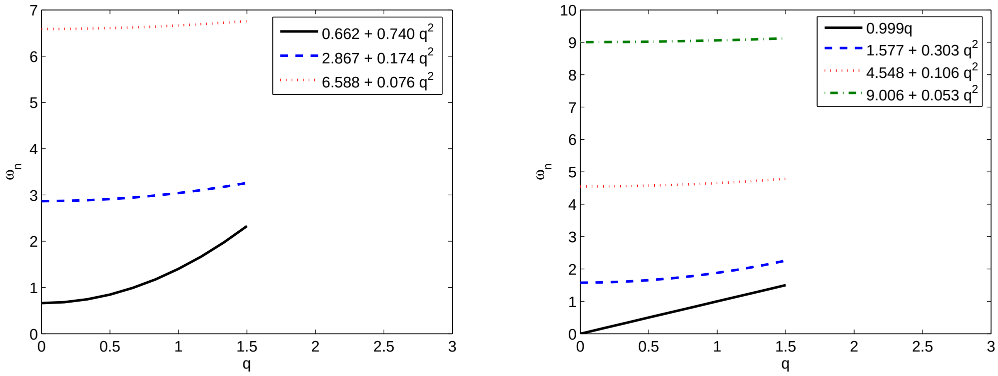

Figure 4: Dispersion relation for vectors (Left) and axial-vectors including the massless pion (Right), $n = 1.26n_0$

The Green's function ($\mathcal{D}_L^{-1}$) may be expanded in terms of the complete set of eigenvalues that diagonalize

$$\mathcal{D}_T(q=0)f = -\left(\frac{K^{-1/3}\Delta}{\omega^2}\right)\lambda f\,, \tag{4.17}$$

by using the eigenvalues

$$(\partial_Z K\Delta\partial_Z)\,\chi_n = -(K^{-1/3}\Delta)\,(\lambda_n^\chi)^2\,\chi_n\ . \tag{4.18}$$

The Green's function is then expanded as

$$\langle Z|\mathcal{D}_T^{-1}(q=0)|Z'\rangle = \sum_{n\in\mathbb{N}}\frac{\chi_n(Z)\chi_n(Z')}{-\omega^2+(\lambda_n^\chi)^2} + \frac{\chi_0(Z)\chi_0(Z')}{-\omega^2}\,, \tag{4.19}$$

which is the same as $\mathcal{D}_L^{-1}$. A rerun of the arguments for the longitudinal current response yields

$$J_T(\omega,q) = 2N\omega\int dZ\,f_T[\mathcal{D}_T^{-1}(f_T\mathcal{V}_T)+\mathcal{V}_T]\,, \tag{4.20}$$

where again the retarded prescription is subsumed and

$$f_T = \frac{K^{-1/3}(\Delta\omega^2-\Delta^{-1}q^2)}{\omega^2}\ . \tag{4.21}$$

In the confined phase, the transverse conductivity follows

$$\sigma_T = 2N\omega\int dZ\,f_T[1+\mathcal{D}_T^{-1}f_T]\ . \tag{4.22}$$

Using the vector meson mode decomposition for $\mathcal{D}_T^{-1}$ (4.7) and the relation (4.10) we can simplify the transverse conductivity

$$\sigma_T = \sigma_L + 2N\omega\int dZ(f_T+f_L)[1+\mathcal{D}_T^{-1}(f_T+f_L)]\ , \tag{4.23}$$

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with

$$f_T + f_L = -\frac{q^2}{\omega^2} K^{-1/3} \Delta^{-1} \ . \tag{4.24}$$

The transverse conductivity is vector meson mediated as shown in figure 2. For $q = 0$, $\sigma_T = \sigma_L$ and vanishes for $\omega \to 0$ in agreement with [12].

5. Quasi-normal mode analysis

we now turn our attention to the deconfined phase of dense holographic models with non-hermitean or absorptive boundary conditions. For that, the retarded prescription on the inversion of $\mathcal{D}_{L,T}$ is best captured by the quasi-normal mode analysis. The latter is enforced analytically by matching for the gapless modes and numerically for the gapped modes. We now present the general formulas pertinent to the longitudinal and transverse currents.

For $\omega \ll 1, q \ll 1$ the equations (3.15) and (3.16) are reduced to

$$\left(\partial_Z \frac{-k_1 k_3 \Delta^3}{\Delta^2\omega^2 + k_3 q^2} \partial_Z\right) a_L = 0 \,, \tag{5.1}$$

$$\left(\partial_Z k_1 k_3 \Delta \partial_Z\right) a_T = 0 \ . \tag{5.2}$$

The general solutions are

$$a_L(Z) = C_L \int_Z^\infty dZ \left(\frac{\omega^2}{-k_1 k_3 \Delta} + \frac{q^2}{-k_1 \Delta^3}\right) \,, \tag{5.3}$$

$$a_T(Z) = C_T \int_Z^\infty dZ \left(\frac{1}{-k_1 k_3 \Delta}\right) = a_L(Z)(q \to 0, \omega \to 1) \,, \tag{5.4}$$

where we imposed the vanishing Dirichlet boundary condition at the boundary $Z = \infty$ i.e. $a_{L/T}(\infty) = 0$. $C_{L/T}$ will be determined by imposing incoming boundary condition at $Z = 0$ which corresponds to the location of matter (confined phase) or the black hole horizon (deconfined phase).

To constrain $C_{L/T}$ we need to know the behavior of $a_{L/T}$ around $Z = 0$. First, we solve the equations (3.15) and (3.16) near $Z = 0$ with fixed ω and q. Second, we take the limit $\omega \ll 1, q \ll 1$. $\Delta_L a_L = 0$ and $\Delta_T a_T = 0$ may be written as

$$a_L'' + \left[\frac{\Delta'(3k_3q^2 + \omega^2\Delta^2(1 - k_3(1/k_3)'\Delta/\Delta'))}{\Delta(k_3q^2 + \omega^2\Delta^2)} + \frac{k_1'}{k_1}\right] a_L' - k_2 \frac{\omega^2\Delta^2 + k_3q^2}{k_3\Delta^2} a_L = 0 \,, \tag{5.5}$$

$$a_T'' + \left[\frac{(k_1k_3\Delta)'}{k_1k_3\Delta}\right] a_T' - k_2 \frac{\omega^2\Delta^2 + k_3q^2}{k_3\Delta^2} a_T = 0 \ . \tag{5.6}$$

In this analysis the variable Z introduced in (3.8) is not convenient due to the complicated form of Δ. To make it simpler we introduce the new variable:

$$z = \frac{1}{(1 + Z^2)^{1/3}}, \tag{5.7}$$

which is nothing but U_0/U in terms of the original coordinate in (3.2). In this coordinate the boundary is $u = 0$ and the horizon or the matter location is $u = 1$.

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5.1 D4/D8: cold and dense

Before analyzing the deconfined phase of D4/D8 with black-hole background absorption, it is amusing to ask whether the D4/D8 confined background with matter in the KK background could be also addressed with *absorptive* or non-hermitean boundary conditions. After all *cold* matter disperses and absorbs waves much like a black-hole. From (5.3) and (5.4) it follows that

$$a_L = C_L(\omega^2 a_\omega(z) - q^2 a_q(z))\,, \tag{5.8}$$

$$a_T = C_T a_\omega(z)\,, \tag{5.9}$$

where

$$a_q(z) \equiv \int_0^z dz' \sqrt{\frac{z'}{(1+d^2 z'^5)^3}}\frac{1}{\sqrt{1-z'^3}}\,, \tag{5.10}$$

$$a_\omega(z) \equiv \int_0^z dz' \sqrt{\frac{z'}{1+d^2 z'^5}}\frac{1}{\sqrt{1-z'^3}}\,, \tag{5.11}$$

for $\omega, q \ll 1$. Recall that the density d is defined by $\frac{3^6\pi^4}{\lambda^2 N_f M_{\rm KK}^3} n_B$ below (3.10). At small densities we expand

$$a_L(z) = C_L\left[(\omega^2-q^2)a^{(0)}(z) - \frac{d^2}{2}(\omega^2-3q^2)a^{(1)}(z) + \frac{3d^4}{8}(\omega^2-5q^2)a^{(2)}(z)\right] + \mathcal{O}(d^6)\,, \tag{5.12}$$

with

$$a^{(0)}(z) = \int_0^z dz'\sqrt{\frac{z'}{1-z'^3}} = -\frac{2}{3}\left(\arcsin(\sqrt{1-z^3}) - \frac{\pi}{2}\right)\,,$$

$$a^{(1)}(z) = \int_0^z dz'\sqrt{\frac{z'}{1-z'^3}}\; z'^5$$

$$= -\frac{1}{60}\sqrt{1-z^3}\left(3\sqrt{z}(7+4z^3) + 7\;{}_2F_1[1/2,5/6;3/2;1-z^3]\right) + \frac{7\sqrt{\pi}}{120}\frac{\Gamma(1/6)}{\Gamma(2/3)}\,,$$

$$a^{(2)}(z) = \int_0^z dz'\sqrt{\frac{z'}{1-z'^3}}\; z'^{10}\,.$$

We don't show $a^{(2)}(z)$ explicitly since it is long and not illuminating.

To impose the *incoming boundary condition* we expand the solution around $z = 1$

$$a_L(z) = C_L A + C_L B\sqrt{1-z} + \mathcal{O}((1-z)^{3/2})\,, \tag{5.13}$$

where

$$A \equiv -\frac{\pi}{3}(q^2-\omega^2) + \frac{7d^2\sqrt{\pi}(3q^2-\omega^2)\Gamma(1/6)}{240\Gamma(2/3)} + \frac{187d^4\sqrt{\pi}(-5q^2+\omega^2)\Gamma(5/6)}{3584\Gamma(4/3)}\,,$$

$$B \equiv \frac{(-8+12d^2-15d^4)q^2 + (8-4d^2+3d^4)\omega^2}{4\sqrt{3}}\,, \tag{5.14}$$

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On the other hand we may first solve the equation near z=1. In general $\Delta_L a_L = 0$ and $\Delta_T a_T = 0$ are

$$a_L'' + \left[\frac{(5d^2z^4)\{3q^2 - \omega^2(1+d^2z^5)\}}{2(1+d^2z^5)\{q^2 - \omega^2(1+d^2z^5)\}} - \frac{3z^2}{2(1-z^3)} - \frac{1}{2z}\right] a_L' - \frac{9\{q^2-\omega^2(1+d^2z^2)\}}{4z(1-z^3)(1+d^2z^5)} a_L = 0\ . \tag{5.15}$$

$$a_T'' + \left[\frac{(5d^2z^4)}{2(1+d^2z^5)} - \frac{3z^2}{2(1-z^3)} - \frac{1}{2z}\right] a_L' - \frac{9\{q^2-\omega^2(1+d^2z^2)\}}{4z(1-z^3)(1+d^2z^5)} a_L = 0\,, \tag{5.16}$$

and reduce to

$$a_{L/T}'' - \frac{1}{2}\frac{1}{1-z} a_{L/T}' + \frac{3}{4}\frac{\omega^2 - \frac{q^2}{1+d^2}}{(1-z)} a_{L/T} = 0\,, \tag{5.17}$$

near $z = 1$. The solutions are

$$a_{L/T} = C_I e^{-i\sqrt{3\gamma(1-z)}}\,, \qquad \gamma \equiv \omega^2 - \frac{q^2}{1+d^2}\,, \tag{5.18}$$

where we imposed the incoming boundary condition at $z = 1$. Their expanded form reads

$$a_L(z) \approx C_I - iC_I\sqrt{3\gamma}\sqrt{1-z} + \mathcal{O}(1-z)\,, \tag{5.19}$$

$$\gamma = \omega^2 - q^2 + q^2d^2 - q^2d^4 + \mathcal{O}(d^6)\ . \tag{5.20}$$

A comparison of (5.13) and (5.19) yields

$$B + iA\sqrt{3\gamma} = 0\,, \tag{5.21}$$

and the dispersion relation is

$$\omega = \pm\left[1 - \frac{d^2}{2} + \frac{3d^4}{8} + \mathcal{O}(d^6)\right] q + \mathcal{O}(d^6)q^2 + \mathcal{O}(d^6)q^3 + \mathcal{O}(q^4)\ . \tag{5.22}$$

The latter resums to

$$\omega = \pm\frac{1}{\sqrt{1+d^2}} q\ . \tag{5.23}$$

This is consistent with the zero mode result obtained in (4.7), (4.11) and figure 4 where we also found the zero mode solution odd in Z. Interestingly enough, the quasinormal mode analysis when applied to the confined and dense KK background with absorptive boundary condition, it yields a massless pole which is the pion pole with a speed $c_\pi = 1/\sqrt{1+d^2}$. Note that there is *no imaginary* part. The reason can be traced back to the + (outgoing) and − (incoming) wave assignment in (5.21), both of which solve

$$0 = B \pm iA\sqrt{3\gamma} = \left[\frac{8 - 4d^2 + 3d^4}{4\sqrt{3}}\sqrt{\gamma} \pm i\sqrt{3}\right]\sqrt{\gamma} = 0\,, \tag{5.24}$$

for $\gamma = 0$.

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For the transverse mode we may follow the same procedure with the substitution in (5.13)

$$a_T(z) = a_L(z)(q \to 0\,, \omega \to 1)\ . \tag{5.25}$$

The relation (5.21) yields

$$\omega = i\left[\frac{2}{\pi} + \left(-\frac{1}{\pi} + \frac{7\Gamma(1/6)}{40\pi^{3/2}\Gamma(2/3)}\right)d^2 + \mathcal{O}(d^4)\right] + \mathcal{O}(q)\,, \tag{5.26}$$

which shows that there is no massless excitation. This channel is indeed gapped in the confined D4/D8 case as we discussed earlier for the case of reflecting boundary conditions.

5.2 D4/D8: hot and dense

The absorptive boundary condition is more appropriate for the deconfined BH background that we now discuss. For that, we rerun the same steps as we did in the previous section. First we solve the equations for $\mathfrak{w} \ll 1, \mathfrak{q} \ll 1$, where $\mathfrak{w} \equiv \frac{\omega}{2\pi T}$ and $\mathfrak{q} \equiv \frac{q}{2\pi T}$. From (5.3) and (5.4)

$$a_L = C_L(\mathfrak{w}^2 a_{\mathfrak{w}}(z) - \mathfrak{q}^2 a_{\mathfrak{q}}(z))\,, \tag{5.27}$$

$$a_T = C_T a_{\mathfrak{w}}(z)\,, \tag{5.28}$$

where

$$a_{\mathfrak{q}}(z) \equiv \int_0^z dz' \sqrt{\frac{z'}{(1+d^2 z'^5)^3}} \tag{5.29}$$

$$= \frac{2}{15} z^{3/2}\left[\frac{3}{\sqrt{1+d^2 z^5}} + 2\ {}_2F_1[3/10, 1/2; 13/10; -d^2 z^5]\right]\,,$$

$$a_{\mathfrak{w}}(z) \equiv \int_0^z dz' \sqrt{\frac{z'}{1+d^2 z'^5}}\frac{1}{1-z'^3}\ . \tag{5.30}$$

Recall that the density d is defined by $\frac{3^5\pi^2}{2^5\lambda N_c N_f T^5 l_s^2} n_B$ below (3.10). To impose the incoming boundary condition we expand a_L around the horizon.

$$\begin{aligned} a_L(1-\epsilon) &= a_L(1) - \epsilon a_L'(1) + \cdots \\ &= C_L \mathfrak{w}^2 a_{\mathfrak{w}}(1) - \frac{C_L \mathfrak{w}^2}{3\sqrt{1+d^2}} - C_L \mathfrak{q}^2 a_{\mathfrak{q}}(1) + \mathcal{O}(\epsilon)\,, \end{aligned} \tag{5.31}$$

where $a_{\mathfrak{w}}(1)$ has a logarithmic divergence and $a_{\mathfrak{q}}(1)$ is finite.

In general $\Delta_L a_L = 0$ and $\Delta_T a_T = 0$ are

$$\begin{aligned} a_L'' + \Bigg[&\frac{(5d^2z^4)\{3(1-z^3)q^2 - \mathfrak{w}^2(1+d^2z^5)\}}{2(1+d^2z^5)\{(1-z^3)\mathfrak{q}^2 - \mathfrak{w}^2(1+d^2z^5)\}} \\ &+ \frac{\mathfrak{w}^2(1+d^2z^5)}{(1-z^3)\mathfrak{q}^2 - \mathfrak{w}^2(1+d^2z^5)}\frac{3z^2}{1-z^3} - \frac{1}{2z}\Bigg]a_L' \\ &- \frac{9\{(1-z^3)\mathfrak{q}^2 - \mathfrak{w}^2(1+d^2z^2)\}}{4z(1-z^3)^2(1+d^2z^5)}a_L = 0\,, \end{aligned} \tag{5.32}$$

$$a_T'' + \left[\frac{(5d^2z^4)}{2(1+d^2z^5)} - \frac{3z^2}{(1-z^3)} - \frac{1}{2z}\right]a_T' - \frac{9\{(1-z^3)\mathfrak{q}^2 - \mathfrak{w}^2(1+d^2z^2)\}}{4z(1-z^3)^2(1+d^2z^5)}a_T = 0\,, \tag{5.33}$$

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which simplify to

$$a''_{L/T} - \frac{1}{1-z} a'_{L/T} + \frac{\mathfrak{w}^2}{4(1-z)^2} a_{L/T} = 0\,, \tag{5.34}$$

near the horizon with $z = 1$. The incoming wave solution near the horizon is

$$a_{L/T} = (1-z)^{-i\mathfrak{w}/2} F(z)\,, \tag{5.35}$$

with $F(z)$ a regular function near $z = 1$ or $z = 1 - \epsilon$. Assuming $\mathfrak{w} \ln \epsilon \ll 1$ we have

$$\begin{aligned} \epsilon^{-i\mathfrak{w}/2} F(1-\epsilon) &= F(1-\epsilon) - \frac{i\mathfrak{w}}{2} \ln \epsilon F(1-\epsilon) + \cdots \\ &= F(1) - \frac{i\mathfrak{w}}{2} \ln \epsilon|_{\epsilon\to 0} F(1) + \mathcal{O}(\epsilon)\ . \end{aligned} \tag{5.36}$$

By comparing the singular part of (5.31) with (5.36) we get

$$C_L = \frac{i3\sqrt{1+d^2}}{2\mathfrak{w}} F(1)\ . \tag{5.37}$$

By comparing the regular part of (5.31) with (5.36) we get the dispersive retation for the longitudinal baryonic waves

$$1 = -\frac{i\mathfrak{w}}{2} - \frac{i3a_{\mathfrak{q}}(1)\sqrt{1+d^2}}{2} \frac{\mathfrak{q}^2}{\mathfrak{w}}\ . \tag{5.38}$$

For small $\mathfrak{w}$ and $\mathfrak{q}$ but fixed $\mathfrak{q}^2/\mathfrak{w}$ the dispersion relation is

$$\begin{aligned} \omega &\approx -\frac{i3\sqrt{1+d^2}}{2} \frac{q^2}{2\pi T} \frac{2}{15} \left[\frac{3}{\sqrt{1+d^2}} + 2\ {}_2F_1[3/10, 1/2; 13/10; -d^2] \right] \\ &\approx -i\frac{q^2}{2\pi T} \left(1 + \frac{2d^2}{13} - \frac{16d^4}{299} + \cdots \right) \quad \text{(For small } d) \\ &\approx -i\frac{q^2}{2\pi T} \left(\frac{2\Gamma(1/5)\Gamma(13/10)}{5\Gamma(1/2)} d^{2/5} + \frac{\Gamma(1/5)\Gamma(13/10)}{5\Gamma(1/2)} d^{-8/5} + \cdots \right) \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(For large } d)\,, \end{aligned} \tag{5.39}$$

where both the small and large baryon density limits are displayed explicitly. The longitudinal diffusion constant is

$$D_L \approx \frac{\sqrt{1+d^2}}{2\pi T} \left[\frac{3/5}{\sqrt{1+d^2}} + \frac{2}{5}\ {}_2F_1[3/10, 1/2; 13/10; -d^2] \right]\ . \tag{5.40}$$

For zero baryon density this is $D = 1/2\pi T$. In the deconfined phase of D4/D8 the baryonic charge diffuses whatever the density. This is expected from baryon number conservation. The presence of the BH in the deconfined phase overwhelms the Fermi effects.

A rerun of the analysis for the transverse baryonic current follows the substitution

$$a_T = a_L(\mathfrak{q} \to 0\,, \mathfrak{w} \to 1)\ . \tag{5.41}$$

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Comparing the singular part of (5.31) and (5.36) gives

$$C_T = \frac{i3\mathfrak{w}\sqrt{1+d^2}}{2}F(1)\,, \tag{5.42}$$

and comparing the regular part of (5.31) and (5.36) yields

$$\mathfrak{w}^3 = i2\ . \tag{5.43}$$

Thus there is no hydrodynamic pole. The transverse baryonic current in dense and deconfined D4/D8 is still gapped. It is much like a *transverse plasmon.*

5.3 D3/D7: hot and dense

For comparison, let us consider in this case the non-chiral and non-confining embedding with D3/D7 at finite temperature and finite density. We consider the massless quark embedding where analytic solutions are available [18]. The induced metric becomes simply $AdS_5 \times S^3$ independent of the gauge field.

$$ds^2 = \frac{Z^2}{R^2}(-f dt^2 + d\vec{x}^2) + f^{-1}\frac{R^2}{Z^2}dZ^2 + R^2 d\Omega_3^2\,, \quad f \equiv 1 - \frac{Z_H^4}{Z^4}\,, \tag{5.44}$$

where $R = 4\pi g_s N_c \alpha'^2$ is the curvature radius. We work in units of $R = 1$. $Z_H = \pi T$ where T is the temperature. SUGRA and SYM quantities will be tied by $\alpha' = 1/\sqrt{\lambda}$ with $\lambda = 4\pi g_s N_c$. With this metric, we compute the DBI action as

$$\begin{aligned} S_{\rm DBI} = -\mathcal{N}\ \mathrm{tr}\int d^4x dZ\ g_{SS}^{3/2}\Bigg[& - g_{00}g_{xx}^3 g_{ZZ} - g_{xx}^3 F_{0Z}F_{0Z} - g_{xx}^2 g_{ZZ}\sum_i F_{0i}F_{0i} \\ & - g_{00}g_{xx}^2\sum_i F_{iZ}F_{iZ} - g_{00}g_{xx}g_{ZZ}\sum_{i>j}F_{ij}F_{ij} - g_{xx}\sum_{i>j}F_{ij}F_{ij}F_{Z0}F_{Z0} + \cdots\Bigg]^{1/2} \end{aligned} \tag{5.45}$$

The result is analogous to the D4/D8 case (3.5) with three differences: 1) $\mathcal{N} = T_7\Omega_3$; 2) there is no contribution from the dilaton; 3) $g_{SS}^{3/2}$ appears instead of $g_{SS}^{4/2}$, since the compact space is S^3 not S^4.

To consider finite baryon density (or chemical potential) we set the background vector U(1) field $\mathbb{A}_0(Z)$ in bulk. Its form follows from minimizing the DBI action (5.45):

$$2\pi\alpha'\mathbb{A}_0' = \frac{d}{\sqrt{Z^6 + d^2}}\ . \tag{5.46}$$

We explicitly recalled $2\pi\alpha'$ and $d \equiv \frac{(2\pi)^3}{\sqrt{\lambda}N_f N_c}n_q$ [18, 17].

Following the analysis in D4/D8 above, we now consider mesonic fluctuations around the density background $\mathbb{A}_0$. In the general form cast in (3.10) the quadratic action reads

$$\begin{aligned} S = N\mathrm{tr}\int d^4x dZ k_1\Bigg[& \Delta^3 F_{Z0}F_{Z0} + \Delta k_2\sum_i F_{0i}F_{0i} \\ & + \Delta k_3\sum_i F_{iZ}F_{iZ} + \Delta^{-1}k_2 k_3\sum_{i>j}F_{ij}F_{ij}\Bigg]\,, \end{aligned} \tag{5.47}$$

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where the information of the background field $\mathbb{A}_0$ is encoded in Δ and

$$N = \frac{\lambda N_f N_c}{2(2\pi)^4}\,, \qquad \Delta = \sqrt{1+d^2 Z^{-6}}\,,$$
$$k_1 = Z^3\,, \qquad k^2 = Z^{-4} f^{-1}\,, \qquad k^3 = f^{-1}, \tag{5.48}$$

as in table. 1.

In this general form we can use (5.1)–(5.6). In terms of the variable $z = \frac{Z_H}{Z}$, $\mathfrak{w} = \frac{\omega}{2\pi T}$, $\mathfrak{q} \equiv \frac{q}{2\pi T}$, and $\mathbf{d} \equiv \frac{d}{(\pi T)^3}$ we have

$$a_L = C_L(\mathfrak{w}^2 a_{\mathfrak{w}}(z) - \mathfrak{q}^2 a_{\mathfrak{q}}(z))\,, \tag{5.49}$$
$$a_T = C_T a_{\mathfrak{w}}(z)\,, \tag{5.50}$$

with

$$a_{\mathfrak{q}}(z) \equiv \int_0^z dz' \frac{z'}{\sqrt{1+\mathbf{d}^2 z'^6}^3} \tag{5.51}$$
$$= z^2(\,{}_2F_1[3/2, 1/3; 4/3, -z^6 d^2])\,,$$
$$a_{\mathfrak{w}}(z) \equiv \int_0^z dz' \frac{z'}{\sqrt{1+\mathbf{d}^2 z'^6}} \frac{1}{1-z'^4}\;. \tag{5.52}$$

Note that the integrand in $a_{\mathfrak{w}}$ exhibits explicitly the BH horizon at $z' = 1$ in units of temperature. At zero temperature the integrand smoothly reduces from $1/(1-z'^4)$ to 1. However, the BH singularity makes the integral logarithmically divergent at the horizon. As a result, the zero temperature limit is singular and will be considered separatly next. To impose the incoming boundary condition we expand a_L around the horizon.

$$a_L(1-\epsilon) = a_L(1) - \epsilon a_L'(1) + \cdots$$
$$= C_L \mathfrak{w}^2 a_{\mathfrak{w}}(1) - \frac{C_L \mathfrak{w}^2}{4\sqrt{1+\mathbf{d}^2}} - C_L \mathfrak{q}^2 a_{\mathfrak{q}}(1) + \mathcal{O}(\epsilon)\,, \tag{5.53}$$

Alternatively, the zero mode equation ($\mathcal{D}_L a_L = 0$) is

$$\partial_z^2 a_L + \left[\frac{3\mathbf{d}^2 z^5}{(1+\mathbf{d}^2 z^6)}\left(\frac{3(1-z^4)\mathfrak{q}^2 - \mathfrak{w}^2(1+\mathbf{d}^2 z^6)}{(1-z^4)\mathfrak{q}^2 - \mathfrak{w}^2(1+\mathbf{d}^2 z^6)}\right) + \frac{\mathfrak{w}^2(1+\mathbf{d}^2 z^6)}{(1-z^4)\mathfrak{q}^2 - \mathfrak{w}^2(1+\mathbf{d}^2 z^6)}\frac{4z^3}{1-z^4} - \frac{1}{z}\right]\partial_z a_L + \frac{4}{1-z^4}\left(\frac{\mathfrak{w}^2}{1-z^4} - \frac{\mathfrak{q}^2}{1+\mathbf{d}^2 z^6}\right) a_L = 0\,, \tag{5.54}$$

$$\partial_z^2 a_T + \left[\frac{3\mathbf{d}^2 z^5}{(1+\mathbf{d}^2 z^6)} - \frac{4z^3}{1-z^4} - \frac{1}{z}\right]\partial_z a_T + \frac{4}{1-z^4}\left(\frac{\mathfrak{w}^2}{1-z^4} - \frac{\mathfrak{q}^2}{1+\mathbf{d}^2 z^6}\right) a_T = 0\,, \tag{5.55}$$

which reduces to

$$a_{L/T}'' - \frac{1}{1-z} a_{L/T}' + \frac{\mathfrak{w}^2}{4(1-z)^2} a_{L/T} = 0\,, \tag{5.56}$$

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near the horizon at $z = 1$. The incoming wave solution is of the form

$$a_{L/T} = (1-z)^{-i\mathfrak{w}/2} F(z)\,, \tag{5.57}$$

with $F(z)$ a regular function near $z = 1$. For $\mathfrak{w} \ln \epsilon \ll 1$ we have

$$\begin{aligned} \epsilon^{-i\mathfrak{w}/2} F(1-\epsilon) &= F(1-\epsilon) - \frac{i\mathfrak{w}}{2} \ln \epsilon F(1-\epsilon) + \cdots \\ &= F(1) - \frac{i\mathfrak{w}}{2} \ln \epsilon|_{\epsilon\to 0} F(1) + \mathcal{O}(\epsilon)\,. \end{aligned} \tag{5.58}$$

A comparison of the singular part of (5.53) and (5.58) yields

$$C_L = \frac{i2\sqrt{1+\mathbf{d}^2}}{\mathfrak{w}} F(1)\,, \tag{5.59}$$

and a comparison of (5.53) and (5.58) yields

$$1 = -\frac{i\mathfrak{w}}{2} - i2a_{\mathfrak{q}}(1)\sqrt{1+\mathbf{d}^2}\frac{\mathfrak{q}^2}{\mathfrak{w}}\,. \tag{5.60}$$

For small $\mathfrak{w}$ and $\mathfrak{q}$ with fixed $\mathfrak{q}^2/\mathfrak{w}$ the dispersion relation follows

$$\begin{aligned} \omega &\approx -i\sqrt{1+\left(\frac{d}{(\pi T)^3}\right)^2}\,\frac{q^2}{2\pi T}\;{}_2F_1[3/2,1/3;4/3;-d^2] \\ &\approx -i\frac{q^2}{2\pi T}\left(1+\frac{1}{8}\left(\frac{d}{(\pi T)^3}\right)^2 - \frac{1}{112}\left(\frac{d}{(\pi T)^3}\right)^4 + \cdots\right) \qquad \left(\text{For small } \frac{d}{(\pi T)^3}\right) \\ &\approx -i\frac{q^2}{2\pi T}\left(\frac{2\Gamma(7/6)\Gamma(4/3)}{\Gamma(1/2)}\left(\frac{d}{(\pi T)^3}\right)^{1/3} + \frac{\Gamma(7/6)\Gamma(4/3)}{\Gamma(1/2)}\left(\frac{d}{(\pi T)^3}\right)^{-5/3} - \cdots\right) \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \left(\text{For Large } \frac{d}{(\pi T)^3}\right)\,, \end{aligned} \tag{5.61}$$

The longitudinal diffusion constant for hot and dense D3/D7 is

$$D_L \approx \frac{1}{2\pi T}\sqrt{1+\left(\frac{d}{(\pi T)^3}\right)^2}\;{}_2F_1[3/2,1/3;4/3;-d^2]\,. \tag{5.62}$$

As mentioned earlier the zero temperature limit is singular owing to the occurence of the BH pole in the issuing integrals.

To analyze the transverse baryonic current[5] in the same limit of small ω, q we follow the same procedure with the substitution in (5.53)

$$a_T = a_L(q \to 0\,, \mathfrak{w} \to 1)\,. \tag{5.63}$$

A comparison of the singular part of (5.53) and (5.58) yields

$$C_T = i2\mathfrak{w}\sqrt{1+\mathbf{d}^2} F(1)\,, \tag{5.64}$$

[5]See [19] for related work on the dispersion relation

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while a comparison of the regular part of (5.53) and (5.58) gives

$$\mathfrak{w}^3 = i2\,, \tag{5.65}$$

which is incompatible with the limits. The transverse baryonic mode in hot and dense D3/D7 is gapped much like the transverse plasmon in dense matter. This reflects on the long-range nature of the transverse forces in holography. We will comment further on this point below.

5.4 D3/D7: cold and dense

In a recent analysis [17] have reported the occurence of a *zero sound* mode in cold D3/D7. For completeness we now rederive their results using our general result (5.3). At zero temperature we set $Z_H = 0$ in (5.44) and change the variable to $z = 1/Z$.

For $\omega \ll 1$ and $q \ll 1$ (5.3) gives

$$a_L = C_L(\omega^2 a_\omega(z) - q^2 a_q(z))\,, \tag{5.66}$$

$$a_T = C_T a_\omega(z)\,, \tag{5.67}$$

with

$$a_q(z) \equiv \int_0^z dz' \frac{z'}{\sqrt{1+d^2 z'^6}^3} = \frac{1}{2} z^2 (\,{}_2F_1[3/2, 1/3; 4/3; -z^6 d^2])\,,$$

$$a_\omega(z) \equiv \int_0^z dz' \frac{z'}{\sqrt{1+d^2 z'^6}} = \frac{1}{2} z^2 (\,{}_2F_1[1/2, 1/3; 4/3; -z^6 d^2])\,, \tag{5.68}$$

Near the horizon a_L is expanded as

$$a_L = C_L A \frac{1}{z} + C_L B + \mathcal{O}\left(1/z^2\right)\,, \tag{5.69}$$

with

$$A \equiv \frac{\omega^2}{d}\,, \qquad B \equiv \frac{(q^2 - 3\omega^2) d^{-2/3} \Gamma(1/3)\Gamma(1/6)}{18\Gamma(1/2)}\,. \tag{5.70}$$

On the other hand the zero mode equations $\mathcal{D}_{L/T} a_{L/T} = 0$ are

$$\partial_z^2 a_L + \left[\frac{3d^2 z^5}{(1+d^2 z^6)}\left(1 + \frac{2q^2}{q^2 - \omega^2(1+d^2 z^6)}\right) - \frac{1}{z}\right]\partial_z a_L + \left(\omega^2 - \frac{q^2}{1+d^2 z^6}\right) a_L = 0\,,$$

$$\partial_z^2 a_T + \left[\frac{3d^2 z^5}{(1+d^2 z^6)} - \frac{1}{z}\right]\partial_z a_T + \left(\omega^2 - \frac{q^2}{1+d^2 z^6}\right) a_T = 0\,. \tag{5.71}$$

Near the horizon the incoming solution is

$$a_L(z) = C_I \frac{e^{iwz}}{z}\,, \tag{5.72}$$

and for $\omega z \ll 1$

$$a_L(z) = \frac{C_I}{z} + i\omega C_I\,, \tag{5.73}$$

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A comparison of (5.69) with (5.73) yields

$$Ai\omega = B\,, \tag{5.74}$$

which yields the dispersion relation reported in [17]

$$\omega = \pm\frac{q}{\sqrt{3}} - \frac{iq^2}{2p\mu_0} + \mathcal{O}(q^3)\,, \tag{5.75}$$

for a massless excitation. Holographic D3/D7 at arbitrarily small temperatures is diffusive. It is not at strictly zero temperature with the occurence of a long-range collective mode. In the next section we suggest that this is a visco-elastic mode, and thereby generalize it to massive quarks. Any amount of temperature (collisions) destroy the Fermi-surface at large λ and large N_c. Indeed, while the temperature effects are of order N_c^0 through the underlying BH metric, the density effects are $1/\lambda$ and $1/N_c$ suppressed through the N_F embeddings either D7 or D8.

Finally and for completeness we note that the transverse baryonic mode follows also from (5.69) with the substitution

$$a_T = a_L(q \to 0\,, \omega \to 1)\ . \tag{5.76}$$

From (5.74) we get

$$\omega = i\frac{d^{1/3}\Gamma(1/3)\Gamma(1/6)}{\Gamma(1/2)}\ . \tag{5.77}$$

The transverse baryonic current is gapped in cold D3/D7 much like the current is plasmon-gapped in a metal.

6. Visco-elastic analysis

The occurence of a collective mode in cold D3/D7 suggests that collectivity through the possible occurence of a Fermi surface at strong coupling maybe at work. To understand that, we propose to understand this collectivity by unifying the hydrodynamical or collision regime with the elastic or collisionless regime.

In figure 5 we show different propagating domains for a wave of frequency ω and momentum q in liquids. The dashed curves are typical wave dispersions. The free particle-hole continuum occupies the lower quadrant. τ is a typical relaxation time to equilibrium, say $\tau \approx 1/T$ (hot) and $\tau \approx 1/\mu$ (cold) for conformal and strongly coupled theories. For waves with $\omega\tau \gg 1$ and large velocities compared to the Fermi velocity v_F of a quasiparticle, we expect *collisionless* wave propagation or *elastic* regime. For waves with $\omega\tau \ll 1$ but still large velocities compared to the Fermi velocity v_F we expect *collision* wave propagation or *hydrodynamic* regime. Typical cold media behave elastically at low temperature and hydrodynamically at higher temperature. Thus, an elastic mode can be turned inelastic by just raising the temperature. This is typically what happens in liquid He^3 where the zero sound transmutes to the first or thermodynamic sound by changing its frequency or temperature to interpolate between the collisionless and collision regimes. To understand these regimes we now introduce a unified visco-elastic framework.

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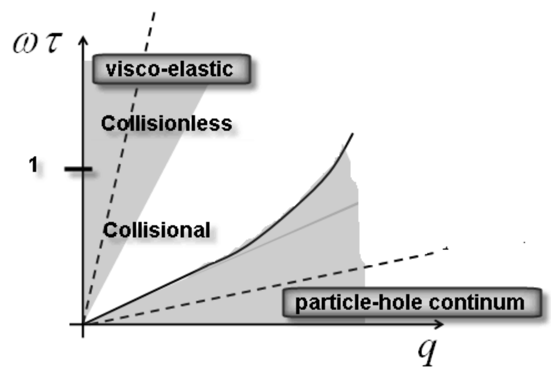

Figure 5: Visco-elastic domain versus the free particle-hole continuum. See text.

6.1 Generalities

In a homogeneous and isotropic solid, the constitutive equations for the elastic displacement field $\vec{u}(t,\vec{x})$ are discussed in the canonical book by Landau and Lifshitz on the theory of elasticity [20]. Specifically

$$m_B\, n_B \frac{\partial^2 \vec{u}}{\partial t^2} = (K + (1-2/p)\,\mu)\,\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) + \mu\nabla^2\vec{u} + \vec{F}(t,\vec{x}) \;, \tag{6.1}$$

where K and μ are the bulk and shear moduli, $p=3$ is the dimensionality of space, $\vec{F}$ is an external volume force, n_B is the baryon equilibrium density and m_B is the bare baryon mass. The massless case more pertinent for D3/D7 will be deduced by inspection below. Without loss of generality, we may write

$$\vec{F}(t,\vec{x}) = n_B \frac{\partial \vec{A}(t,\vec{x})}{\partial t} \;, \tag{6.2}$$

which is the 'baryon electric field'. Since the transverse part of $\vec{A}$ induces a 'baryon magnetic field' we expect (6.2) to also include the magnetic contribution as it plays the role of the Lorentz force. Since we are interested in the induced baryon current through $\vec{A}$, the magnetic effects are second order and will be omitted. The baryon current density is

$$\vec{j}(t,\vec{x}) = n_B \frac{\partial \vec{u}(t,\vec{x})}{\partial t} \;. \tag{6.3}$$

Inserting (6.2) and (6.3) in (6.1) and taking the Fourier transforms yield

$$\begin{aligned} -i\omega\, m_B \vec{j}(\omega,\vec{q}) = &\left(\frac{K}{n_B} + \left(1-\frac{2}{p}\right)\frac{\mu}{n_B}\right)\frac{\vec{q}\,(\vec{q}\cdot\vec{j}(\omega,\vec{q}))}{i\omega} \\ &+ \frac{\mu}{n_B}\frac{q^2}{i\omega}\,\vec{j}(\omega,\vec{q}) - i\omega\, n_B \vec{A}(\omega,\vec{q}) \;. \end{aligned} \tag{6.4}$$

Decomposing the current $j = j_T + j_L$ and the potential $A = A_L + A_T$ along $\vec{q}$ and transverse to $\vec{q}$ allows the transverse and longitudinal induced currents

$$\vec{j}_T(\omega,\vec{q}) = \frac{n_B/m_B}{1 - \frac{\mu}{n_B^2}\frac{n_B q^2}{m_B\omega^2}}\,\vec{A}_T(\omega,\vec{q}) \,, \tag{6.5}$$

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and

$$\vec{j}_L(\omega,\vec{q}) = \frac{n_B/m_B}{1-\left(\frac{K}{n_B^2}+2\left(1-\frac{1}{p}\right)\frac{\mu}{n_B^2}\right)\frac{n_B q^2}{m_B\omega^2}}\,\vec{A}_L(\omega,\vec{q})\ . \tag{6.6}$$

j_L relates directly to the induced baryon density through the local conservation law as qj_L/ω.

The response currents (6.5) and (6.6) have a direct analogy with their counterparts in a liquid. Indeed, using hydrodynamics for the baryon current density in a liquid we can write the analogue of (6.4). In the linear response approximation

$$\begin{aligned} -i\omega\, m_B\vec{j}(\omega,\vec{q}) = &\left(\frac{K}{n_B}-\frac{i\omega\zeta}{n_B}+\left(1-\frac{2}{p}\right)\frac{i\omega\eta}{n_B}\right)\frac{\vec{q}\,(\vec{q}\cdot\vec{j}(\omega,\vec{q}))}{i\omega} \\ &+\frac{i\omega\eta}{n_B}\frac{q^2}{i\omega}\,\vec{j}(\omega,\vec{q})-i\omega\, n_B\vec{A}(\omega,\vec{q})\ , \end{aligned} \tag{6.7}$$

where the hydrostatic pressure term $\mathbf{p}$ in the Euler equation was traded with the longitudinal baryon current through the continuity equation,

$$-\vec{\nabla}\mathbf{p}(\omega,\vec{q}) = -\frac{\partial\mathbf{p}}{\partial n}\nabla n_B(\omega,\vec{q}) = \frac{\partial\mathbf{p}}{\partial n}\vec{q}\left(\frac{\vec{q}\cdot\vec{j}(\omega,\vec{q})}{i\omega}\right)\ , \tag{6.8}$$

where η and ζ are the shear and bulk viscosities; $\mathbf{p}$ is the equilibrium pressure as a function of density with $K=n_B\partial\mathbf{p}/\partial n$ the bulk modulus. (6.4) is very similar to (6.7) except for: 1/ the shear modulus in the solid becomes purely imaginary or $-i\omega\eta$ in the liquid; 2/ the bulk modulus acquires an imaginary part through $-i\omega\zeta$ in the liquid. Both imaginary parts vanish at $\omega=0$ making the liquid insensitive to shear at zero frequency. This also means that their contributions in $j_{L,T}$ are diffusive.

The solid and liquid visco-elastic coefficients can be described in a unified manner through (6.5) and (6.6) by substituting

$$K\to\mathbf{K}(\omega)\ =\ K(\omega)-i\omega\zeta(\omega)\,, \tag{6.9}$$

$$\mu\to\mathbf{M}(\omega)\ =\ \mu(\omega)-i\omega\eta(\omega)\,, \tag{6.10}$$

as the complex and frequency dependent bulk $\mathbf{K}$ and shear $\mathbf{M}$ visco-elastic coefficients. The solid has real $\mathbf{M}$ with a small imaginary part η (zero up to the uncertainty principle) while the liquid has imaginary $\mathbf{M}$ with a small μ and large imaginary part η. The bulk modulus K is about the same in liquid and solid, and of the same order of magnitude as the shear viscosity η.

In light of (6.10) it follows from (6.6) and (6.5) that the longitudinal current admits a gapless pole (compression mode) at

$$\omega\approx\left(\frac{K}{m_Bn_B}+2\left(1-\frac{1}{p}\right)\frac{\mu}{m_Bn_B}\right)^{1/2}q-i\left(\frac{\zeta}{m_Bn_B}+2\left(1-\frac{1}{p}\right)\frac{\eta}{m_Bn_B}\right)\frac{q^2}{2}\,, \tag{6.11}$$

while the transverse current admits a gapless pole (shear mode) at

$$\omega\approx\sqrt{\frac{\mu}{m_Bn_B}}q-i\frac{\eta}{m_Bn_B}\frac{q^2}{2}\ . \tag{6.12}$$

We see that for finite frequency waves and in the long-wavelength approximation the way a solid responds to external wave-stress is similar to the way a liquid does. The difference is that in a solid $\mu \approx K$ and ζ, η are small, while in a liquid μ is close to zero and ζ, η are large. In the liquid the transverse or shear mode becomes diffusive.

6.2 Cold D3/D7

D3/D7 at finite density yields a gapped transverse baryonic current and a gapless longitudinal baryonic current [17]. The gapless longitudinal baryonic current can be compared with the longitudinal visco-elastic mode (6.11) with

$$\left(\frac{K}{m_B n_B} + 2\left(1 - \frac{1}{p}\right)\frac{\mu}{m_B n_B}\right)^{1/2} \Leftrightarrow \frac{1}{\sqrt{p}}, \tag{6.13}$$

$$\left(\frac{\zeta}{m_B n_B} + 2\left(1 - \frac{1}{p}\right)\frac{\eta}{m_B n_B}\right) \Leftrightarrow \frac{1}{p\mu_B}. \tag{6.14}$$

The compressibility is readily tied with the equation of state for any embedding

$$\frac{K}{m_B n_B} \equiv \left(\frac{\partial P}{\partial \epsilon}\right)_S = \frac{1}{p}, \tag{6.15}$$

since $\epsilon - pP = 0$ in a conformal theory.[6] The energy momentum tensor is still traceless at finite temperature and density for massless fermions. The gapless and longitudinal baryonic mode has the speed of the first sound $c_1 = 1/\sqrt{p}$ for zero shear modulus $\mu = 0$ and massless quarks. For massive quarks $\epsilon - pP \neq 0$. For D3/D7 it follows from [18] that

$$\epsilon = \frac{1}{4}\gamma N(2\pi\alpha')^4(\mu_q^2 - m_q^2)(3\mu_q^2 + m_q^2), \tag{6.16}$$

$$P = \frac{1}{4}\gamma N(2\pi\alpha')^4(\mu_q^2 - m_q^2)^2, \tag{6.17}$$

where $\gamma \approx 0.363$. The visco-elastic mode has a speed

$$c_1 = \sqrt{\left(\frac{\partial P}{\partial \epsilon}\right)_S} = \sqrt{\frac{\mu_q^2 - m_q^2}{3\mu_q^2 - m_q^2}}, \tag{6.18}$$

in agreement with the detailed quasi-normal mode analysis in [21].

Since the mode $\omega = q/\sqrt{3}$ lies within the free particle-hole continuum as shown in figure 2, it is susceptible to Landau-like damping through single particle-hole or multi-particle-multi-hole. Again, the visco-elastic analysis suggests that the bulk viscosity in cold D3/D7 with massless quarks is

$$\frac{\eta}{n_B} = \frac{\hbar}{2(p-1)}, \tag{6.19}$$

where we used (A.2) and (A.3). In conformal theories the shear modulus ($\mu = 0$) and bulk viscosity are zero (ζ). For $p = 3$, we have $\eta/n_B = \hbar/4$, where $\hbar$ has been restored.

[6] p is the dimensionality of space and P is pressure.

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This is to be compared with $\hbar/6\pi$ argued in [22] using the Stokes-Einstein relation and the uncertainty principle for cold and strongly coupled Fermions.

It is worth noting that (6.19) can be rewritten as

$$\frac{\eta}{n_B} = \left(\frac{p}{p-1}\right)\frac{\hbar}{2p} = \frac{n_F k_F}{N_c N_f n_B}\frac{\hbar}{p}\,, \tag{6.20}$$

where

$$\frac{n_F}{n_B} = \frac{N_c N_f \int_0^{k_F} \frac{d^p k}{(2\pi)^p}\frac{1}{2k}}{\int_0^{k_F} \frac{d^p k}{(2\pi)^p}} = \frac{N_c N_f}{2k_F}\frac{p}{p-1}\,, \tag{6.21}$$

is the ratio of the quark density at the Fermi surface normalized to the baryon density. For massless quarks $n_B k_F = \epsilon + P$ with $n_q = N_c N_f n_B$ so that in general

$$\frac{\eta}{\epsilon + P} = \frac{n_F}{n_q}\frac{\hbar}{p}\,. \tag{6.22}$$

This result can generalized to finite mass by noting that at zero temperature $\epsilon + P = n_B \mu_B$ and substituting $2k \to 2\sqrt{k^2 + m_q^2}$ in n_F in (6.21). Specifically,

$$\frac{n_F}{n_B} = \frac{N_c N_f \int_0^{k_F} \frac{d^p k}{(2\pi)^p}\frac{1}{2\sqrt{k^2+m_q^2}}}{\int_0^{k_F} \frac{d^p k}{(2\pi)^p}} = \frac{N_c N_f v_F}{4k_F}\ {}_2F_1(1/2, p/2; 1+p/2; -v_F^2)\,, \tag{6.23}$$

with $v_F = k_F/m_q$. Thus

$$\frac{\eta}{n_B} = \left(\frac{\mu_B v_F}{4k_F}\right)\ {}_2F_1(1/2, p/2; 1+p/2; -v_F^2)\ \frac{\hbar}{p}\,. \tag{6.24}$$

The visco-elastic (5.75) for massless quarks, turn to

$$\omega = c_1 q - \frac{i}{p}\left(1 - \frac{1}{p}\right)\left(\frac{v_F}{k_F}\ {}_2F_1(1/2, p/2; 1+p/2; -v_F^2)\right)\frac{q^2}{4}\,, \tag{6.25}$$

for massive quarks. c_1 is the first sound speed. For $D3/D7$ it is explicitly given in (6.18), while for arbitrary Dp/Dq it follows from the known equations of state [18].

In D3/D7 any infinitesimal temperature washes out the Fermi surface, resulting in a diffusive baryonic phase. Thermal collisions at strong coupling take over the collisions through the Fermi surface however small is the temperature. As noted earlier, this is the hallmark of strong coupling holography whereby the BH contribution is of order N_c^0 while the Fermi contributions are $1/N_c$ suppressed.

7. Random phase approximation

If cold D3/D7 exhibits collectivity at large λ and large N_c that is consistent with a constitutive visco-elastic analysis, why the cold D4/D8 results above are all gapped. The short answer is that in D4/D8 the baryons are solitons, so baryonic motion with a Pauli-blocked

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Fermi surface is subleading in $1/N_c$ as shown in figure 2. Baryons move semi-classically by quantizing the isorotations and rotations both of which are $1/N_c$ suppressed. To estimate some of these contributions we note that the baryons in D4/D8 are flavored instantons with core sizes of order $1/\sqrt{\lambda}$. They are heavy with $m_B = 8\pi^2 N_c \lambda$. So the semiclassical descriptive of the translational zero modes follow from the point-like effective action

$$S = \phi^+ \left(i\partial_t - \frac{(-i\vec{\nabla} - \vec{\mathcal{V}})^2}{2m_B} - \mu_B \right) \phi - \frac{1}{2}\alpha(\phi^+\phi)^2 + \ldots, \tag{7.1}$$

where the repulsive interaction $\alpha = (24\pi^4/4M_{KK}^2)(N_c/\lambda)$ is set to reproduce the energy density and pressure of the holographic matter (2.8) with $p \approx \alpha n_B^2/2$. The dotted contributions involve higher derivative terms, e.g. $(\phi^+(-i\nabla - \mathcal{V})/m_B\phi)^2$ which are suppressed by $1/N_c$. Again, note that the limit N_c and λ large do not commute for α. For fixed λ and large N_c the repulsion is strong as it should, while for fixed N_c and large λ the repulsion is weak. Here $\vec{\mathcal{V}}$ is the probing baryonic vector source. In large N_c the baryons are scalar fermions with no assigned spin to leading order in $1/N_c$. Therefore their spin degeneracy is 1.

In the RPA approximation, the zero modes in (7.1) integrate to the effective action

$$S_{RPA}(\mathcal{V}) = \frac{1}{2}\mathcal{V}_L\Delta_L\mathcal{V}_L + \frac{1}{2}\mathcal{V}_T\Delta_T\mathcal{V}_T\,, \tag{7.2}$$

with $j_L = \Delta_L\mathcal{V}_L$ and $j_T = \Delta_T\mathcal{V}_T$ for the longitudinal and transverse currents. $\omega\Delta_{L,T}$ are the longitudinal and transverse baryonic conductivities respectively. The RPA contributions as shown in figure 6 resum to

$$\begin{aligned}\Delta_L &= \frac{\Pi_L}{1 - \alpha\frac{q^2}{\omega^2}\Pi_L}\,,\\ \Delta_T &= \frac{\Pi_T}{1 - \tilde{\alpha}\frac{q^2}{\omega^2}\Pi_T}\,,\end{aligned} \tag{7.3}$$

with

$$\Pi_{L,T} = \sum_k^F \frac{1}{m_B} + \sum_k^F \left(\frac{k_{L,T}}{m_B}\right)^2 \Delta_F(k+q)\Delta_F(k)\ . \tag{7.4}$$

The solid lines in figure 6 lie in the Fermi surface. $\tilde{\alpha}/\alpha \approx 1/N_c$. The transverse contributions follow solely from the dotted terms in (7.1). The summation is carried over the Fermi surface. Δ_F is the massive and non-relativistic fermion propagator associated to (7.1) in the presence of a Fermi surface. The first contribution in (7.4) is from the seagull term in (7.1) and the second contribution is from the particle-hole bubble. The londitudinal vector response Δ_L relates to the scalar density-density response function by current conservation. Specifically, $\Pi_L = \omega^2/q^2\Pi$ where

$$\Pi(q) = \sum_k^F \Delta_F(k+q)\Delta_F(k) \tag{7.5}$$

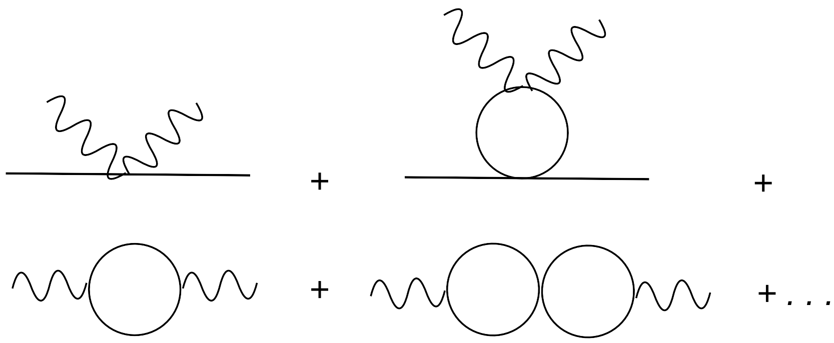

Figure 6: RPA contributions following from (7.1).

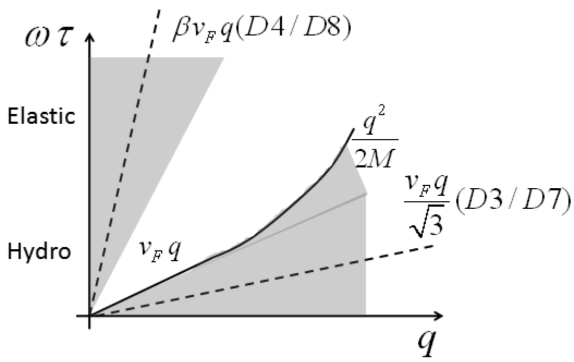

Figure 7: Dispersion relations (dotted lines) for D4/D8 and D3/D7.

is the Lindhard function for scalar fermions. For $\omega, q \to 0$ but fixed $\beta = \omega/q/v_F$ it takes the form

$$\Pi(q) \approx \frac{m_B k_F}{2\pi^2}\left(-1 + \frac{\beta}{2}\ln\left|\frac{1+\beta}{1-\beta}\right|\right) - i\theta(1-|\beta|)\frac{m_B \beta k_F}{4\pi} \; . \tag{7.6}$$

The quasiparticle spectrum following from (7.5) is schematically displayed in figure 7 with massless quasiparticles of energy $\omega = v_F q$ at small k, and free massive fermions with $\omega = q^2/2m_B$ away from the Fermi surface. This description is rooted in weak coupling.

We note that the longitudinal current in (7.3) develop a *massless* poles for strongly repulsive fermions as D4/D8. The longitudinal modes are directly tied to the Lindhard function by current conservation. They follow from $1 = \alpha\Pi$. In particular the longitudinal sound modes is stable for $\beta > 1$ above the quasiparticle cut (no imaginary part) with a speed ($\beta \gg 1$)

$$c_L = \beta\, v_F \approx \sqrt{\frac{\alpha m_B k_F}{3\pi^2}}\, v_F = \sqrt{\alpha n_B/m_B} = c_1 \, , \tag{7.7}$$

in agreement with the pressure $p \approx \alpha n_B^2/2$ in leading order in the density. For $\beta > 1$ the decay of the longitudinal baryonic in D4/D8 is not through Landau-like damping.

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The transverse mode follows from the pole at $\omega^2 = \tilde{\alpha} q^2 \Pi_T$. For $\tilde{\alpha} \approx \alpha_*/(N_c q^2)$ the transverse mode is gapped since

$$\Pi_T(q) \approx \frac{n_B}{m_B} + \frac{n_B}{20 m_B \beta^2} - i\theta(1-|\beta|)\frac{3\pi n_B}{8 m_B}, \tag{7.8}$$

for $\omega, q \to 0$ and fixed $\beta = \omega/q/v_F$. The transverse gap is typically $\omega_T \approx \alpha_* n_B/m_B N_c$ with α_* following from a a numerical analysis of the transverse quasi-normal modes in holography. In light of the RPA analysis, the holographic result *suggests* that the transverse baryonic fluctuations are long ranged and unscreened unlike their longitudinal counterparts.

8. Conclusions

We have analyzed the baryonic transport in D4/D8 (chiral) and D3/D7 (nonchiral) at finite density and/or temperature. D4/D8 is a holographic model of QCD at large N_c and large 'tHooft coupling λ. The transverse baryonic current in D4/D8 is saturated by the medium modified vector mesons in the confined phase with $T < M_{KK}/\pi$. The vector spectrum is gapped since matter is uncompressible at large N_c. Confined D4/D8 matter becomes compressible to order $1/N_c$ with the occurence of a gapless longitudinal vector mode. While $1/N_c$ effects are difficult to assess in holography, we have provided an RPA argument for the speed of the gapless mode using an effective action for baryons constrained by holography. D4/D8 is diffusive in the deconfined regime.

D3/D7 is diffusive at all temperatures except zero where it is visco-elastic. This is a hallmark of holography at large N_c and large λ. Indeed, the temperature effects are mediated by the BH background and leading in $1/N_c$, while the baryonic density effects are carried by the probe branes which are N_f/N_c suppressed. At strong coupling and large N_c the thermal or collisional collision regime is dominant. The exception is D3/D7 at zero temperature but finite density as recently pointed by [17]. A Fermi surface (albeit strongly coupled) maybe at work in this case that suggests a visco-elastic regime. A longitudinal gapless mode emerges with a small width suggestive of a shear viscosity to baryon ratio $\eta/n_B = \hbar/4$ in cold but dense $D3/D7$. This mode is turned diffusive by arbitrarily small temperatures at strong coupling. Our observations extend readily to massive quarks in D3/D7.

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A. Cold Dp/Dq

It is interesting to analyze the equation of state of cold Dp/Dq embeddings. We consider Dq probe branes whose worldvolume spans an AdS_{p+2} factor and wraps the n-sphere S^n in $AdS_5 \times S^5$, where $p = q - n - 1$. For example, $q = 5, p = 2$ corresponds to $D5$ branes on $AdS_4 \times S^2$, and $q = 3, p = 1$ corresponds to $D3$ branes on $AdS_3 \times S^1$ in $AdS_5 \times S^5$.

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At zero temperature and for massless quarks, the pressure P and energy density ϵ read [17]

$$P = -\frac{\Omega}{V_p} = \frac{1}{p+1}\frac{\alpha}{\mathcal{N}_q^{1/p}}\widetilde{n}_B^{\frac{p+1}{p}}\,, \tag{A.1}$$

$$\widetilde{\mu}_B = \frac{\alpha}{\mathcal{N}_q^{1/p}}\widetilde{n}_B^{\frac{1}{p}}\,, \tag{A.2}$$

$$\epsilon = -P + \widetilde{\mu}_B\widetilde{n}_B = \frac{p}{p+1}\frac{\alpha}{\mathcal{N}_q^{1/p}}\widetilde{n}_B^{\frac{p+1}{p}}\,, \tag{A.3}$$

$$\frac{\epsilon}{\epsilon_0} = \frac{\alpha}{4\pi^{3/2}}\left(\frac{1}{\Gamma(p/2+1)\pi N_c N_f}\right)^{1/p}\left(\frac{\lambda^{\frac{p+1}{2}}}{\mathcal{N}_q}\right)^{1/p}\,, \tag{A.4}$$

$$\epsilon_0 = 2\sqrt{\pi}\left(\frac{\Gamma(p/2)p}{4N_cN_f}\right)^{1/p}\frac{p}{p+1}n_q^{(p+1)/p}\,, \tag{A.5}$$

where $\widetilde{n}_B \equiv \frac{\sqrt{\lambda}}{2\pi}n_q$, $\widetilde{\mu}_B \equiv \frac{2\pi}{\sqrt{\lambda}}\mu_q$, and $\alpha \equiv \frac{\Gamma(1/2-1/2p)\Gamma(1+1/2p)}{\Gamma(1/2)}$. $\mathcal{N}_q \equiv N_f T_{Dq} V_n$ with V_n the volume of a unit n-sphere and T_{Dq} is the Dq brane tension [18]. That is $N_7 = \lambda N_f N_c/(2\pi)^4$, $N_5 = \frac{N_f N_c\sqrt{\lambda}}{2\pi^3}$, and $N_3 = \frac{N_f N_c}{\pi}$. The ratio ϵ/ϵ_0 shows the energy density in cold Dp/Dq normalized to the free Fermi energy density:

$$\frac{\epsilon}{\epsilon_0} \approx \lambda^{1/p}\,. \tag{A.6}$$

For $Dp/Dq \equiv (D3/D7), (D2/D5), (D1/D3)$ dense Dp/Dq is unbound at large λ.

B. Fermionic drag

B.1 Hot D3/D7

Hot D3/D7 is diffusive at all temperatures. The drag coefficient η_D is inversely proportional to the baryonic diffusion constant D_q through the Einstein formulae at strong coupling [22, 23]

$$\eta_D = \frac{T}{D_q}\,, \tag{B.1}$$

and for slowly moving particles. In equilibrium, the diffusion constant ties with the baryonic conductivity σ_q (at zero frequency and momentum)

$$\Xi = \frac{<(\Delta N)^2>}{TV_3} = \frac{\sigma_q}{D_q}\,, \tag{B.2}$$

where Ξ is the baryonic susceptibility. Thus

$$\eta_D = \frac{\Xi}{\sigma_q}T\,. \tag{B.3}$$

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The conductivity σ_q has been obtained using Ohm's law in [24] (See also next appendix)

$$\sigma_q = \frac{N_c N_f T}{4\pi}\sqrt{c^6 + \mathbf{d}^2}\,, \quad \mathbf{d} \equiv \frac{8n_q}{\sqrt{\lambda}N_c N_f T^3}\,, \tag{B.4}$$

with $c \equiv \cos^6\theta(z_*)$ and $z_* \equiv 4/(\pi^2 T^2)$ a dynamically generated value of a scalar profile at the BH horizon. Massless quarks correspond to $\theta = 0$ and infinite mass quarks to $\theta = \pi/2$. Thus

$$\eta_D = \frac{\Xi}{N_c N_f}\frac{4\pi}{\sqrt{c^6 + \mathbf{d}^2}}\,. \tag{B.5}$$

This expression expresses the drag of a quark in diffusive D3/D7 for arbitrary mass, temperature and baryon density. When $m_q = 0$

$$\eta_D = \frac{\Xi}{N_c N_f}\frac{4\pi}{\sqrt{1 + \mathbf{d}^2}} = \frac{2\pi T^2}{\sqrt{1 + \mathbf{d}^2}}\,, \tag{B.6}$$

where $\Xi = \frac{N_f N_c T^2}{2}$ [25]. When $m_q = \infty$

$$\eta_D = \frac{\Xi}{N_c N_f}\frac{4\pi}{\mathbf{d}} = \frac{\Xi}{n_q}\frac{\pi}{2}\sqrt{\lambda}T^3\,, \tag{B.7}$$

where Ξ can be read off from eq. (5.7) in [25].

B.2 Cold D3/D7

The zero temperature case is visco-elastic as we suggested earlier. In this regime the fermionic conductivity ties to the diffusion constant by the Kubo formulae

$$\sigma_q = n_F D_q\,, \tag{B.8}$$

$$n_F = N_c N_f \int \frac{d^p k}{(2\pi)^p}\frac{1}{2E_k}(f(E_k) + \bar{f}(E_k))\,, \tag{B.9}$$

where f is Boltzmann distribution function and $E_k = \sqrt{k^2 + m_q^2}$. This relation follows from the relaxation time approximation in the quark probe phase space irrespective of strong or weak coupling. Relaxation to equilibrium at strong coupling is subsumed. For infinitesimal temperatures and for finite quark mass [24, 26] (see also next appendix)

$$\frac{\sigma_q}{T} = \frac{N_c N_f}{4\pi}\sqrt{c^6 + \mathbf{d}^2}\,. \tag{B.10}$$

From (B.1) it follows that the drag is

$$\frac{\eta_D}{n_B} = \frac{n_F}{N_c N_f n_B}\frac{4\pi}{\sqrt{c^6 + \mathbf{d}^2}}\,. \tag{B.11}$$

This is the general form of the quark drag in a cold holographic medium (Coulomb phase) with *infinitesimal* temperature. We will assume it also for $T = 0$ by continuity.

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For m_q finite ($c \neq 0$) and $N_c, \lambda \to \infty$ ($\mathbf{d} \to 0$)

$$\eta_D \approx \frac{n_F}{N_c N_f} \frac{4\pi}{c^3} \,. \tag{B.12}$$

When $T = 0$ and $m_q = 0 (c = 1)$

$$\eta_D = \frac{2^{(2-p)} \pi^{(1-p/2)}}{\Gamma(p/2)} \frac{\mu_q^{p-1}}{p-1} \,, \tag{B.13}$$

where we used

$$n_F = N_c N_f \int_0^{\mu_q} \frac{d^p k}{(2\pi)^p} \frac{1}{2k} \,. \tag{B.14}$$

When $T = 0$ and $m_q \neq 0 (c \neq 1)$ and $p = 3$

$$\frac{\eta_D(m_q)}{\eta_D(m_q = 0)} = \left[\frac{\mu_q}{\sqrt{\mu_q^2 - m_q^2}} - \frac{m_q^2}{\mu_q^2 - m_q^2} \ln \left(\frac{1 + \frac{\mu_q}{\sqrt{\mu_q^2 - m_q^2}}}{\frac{m}{\sqrt{\mu_q^2 - m_q^2}}} \right) \right] \frac{1}{c^3} \,, \tag{B.15}$$

where $n_q = \sqrt{m_q^2 + k_F^2}$.

For $m_q \to \infty (c = 0)$

$$\eta_D \approx \frac{n_F}{N_c N_f} \frac{4\pi}{\mathbf{d}} = \frac{n_F}{n_q} \frac{\pi\sqrt{\lambda}}{2} T^3 = \frac{1}{2(p-2)} \pi \sqrt{\lambda} T^2 \,, \tag{B.16}$$

where $n_q = N_c N_f n_B$ and

$$\frac{n_F}{n_q} = \frac{\int d^p k \frac{1}{k^2/2m_q} e^{-k^2/2m_q T}}{\int d^p k e^{-k^2/2m_q T}} = \frac{1}{p-2} \frac{1}{T} \,. \tag{B.17}$$

For $p = 3$, η_D is the drag coefficient reported in [23].

C. Baryonic conductivity

The baryonic conductivity σ_q in D3/D7 has been derived by various methods [27, 24, 26]. Generically, the Kubo formulae for the conductivity is

$$\sigma_q = - \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G_{xx}^{\mathrm{ret}}(K) \Big|_{\omega = |\vec{k}|} \,, \tag{C.1}$$

where only the transverse response function contributes, as the longitudinal part vanishes for light-like momenta by charge conservation. For a rotationally symmetric medium, $G_{xx} = G_{yy} = G_{zz}$ are the components of the $j_x j_x$, $j_y j_y$ and $j_z j_z$ retarded baryonic current correlations.

Using AdS/CFT the transverse response can be extracted from (5.55), i.e.

$$a_T'' - \frac{u(u + \mathbf{d}^2 u(-3 + 7u^2))}{2(1-u^2)(1 + \mathbf{d}^2 u^3)} a_T' + \frac{\mathfrak{w}^2 u}{(1-u^2)^2} \frac{1 + u\mathbf{d}^2}{1 + u^3 \mathbf{d}^2} a_T = 0 \,, \tag{C.2}$$

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with $u \equiv z^2$. The horizon($u = 1$) is a regular singular point and the solution behaves as $a_T \sim (1-u)^{\pm i\mathfrak{w}/2}$. We choose the incoming boundary condition($a_T \sim (1-u)^{-i\mathfrak{w}/2}$) and extract the singularity at $u = 1$ by substituting

$$a_T = (1-u^2)^{-i\mathfrak{w}/2} F(u) \,. \tag{C.3}$$

$F(u)$ is regular at $u = 1$ and satisfies the following equation

$$\begin{aligned} F'' &+ \frac{u(-4+u\mathbf{d}^2(3-7u^2)) + 2i(1+u)(1+\mathbf{d}^2u^3)\mathfrak{w}}{2(1-u^2)(1+\mathbf{d}^2u^3)} F' \\ &+ \frac{i(1+u)(2+\mathbf{d}^2u^2(3+5u))\mathfrak{w} + (-1+u+\mathbf{d}^2u^2(4+3u+u^2))\mathfrak{w}^2}{4(1-u^2)(1+u)(1+\mathbf{d}^2u^3)} F = 0 \,. \end{aligned} \tag{C.4}$$

In the hydrodynamic region($\mathfrak{w} \ll 1$) we may expand $F(u)$ in terms of $\mathfrak{w}$ as

$$F = F_0 + \mathfrak{w} F_1 + \mathfrak{w}^2 F_2 \cdots \,, \tag{C.5}$$

and unwind $F_0, F_1, F_2, \cdots$ order by order. For that consider the case with $\mathbf{d} = 0$. The zeroth order equation is solved with

$$F_0' = \frac{C}{1-u^2} \,, \tag{C.6}$$

where C is an integration constant. Regularity at $u = 1$ forces $C = 0$. So F_0 is a constant. At next order we have

$$F_1' = \frac{i}{2(-1+u^2)} F_0 + \frac{C}{-1+u^2} \,. \tag{C.7}$$

Again regularity at $u = 1$ sets the constant $C = -iF_0/2$. Thus

$$F_1' = \frac{iF_0}{2(1+u)} \,, \tag{C.8}$$

which is enough unwinding of the transverse solution for the Green's function in the zero frequency limit needed for the Kubo formulae.

To obtain the retarded Green's function we need the boundary action

$$\begin{aligned} S &= \lim_{Z\to\infty} -2\widetilde{N} \int \frac{d\omega dq}{(2\pi)^2} \Delta k_1 k_3 \frac{1}{\mathfrak{w}^2} a_T'(Z,\mathfrak{w}) a_T(Z,-\mathfrak{w}) \\ &= \lim_{u\to 0} -4\widetilde{N}(\pi T)^2 \int \frac{d\omega dq}{(2\pi)^2} \frac{1}{\mathfrak{w}^2} a_T'(u,\mathfrak{w}) a_T(u,-\mathfrak{w}) \\ &= -4\widetilde{N}(\pi T)^2 \int \frac{d\omega dq}{(2\pi)^2} \frac{1}{\mathfrak{w}^2} F'(0,\mathfrak{w}) F(0,-\mathfrak{w}) \,, \end{aligned}$$

where we recovered $2\pi\alpha'$ and $\widetilde{N} \equiv N(2\pi\alpha')^2 = \frac{N_c N_f}{2(2\pi)^2}$. The retarded Green function is then

$$\begin{aligned} G^{\rm ret}_{xx} &= \frac{\delta^2 S}{\delta a_x(0,\mathfrak{w}) \delta a_x(0,-\mathfrak{w})} = \frac{\mathfrak{w}^2 \delta^2 S}{\delta F(0,\mathfrak{w}) \delta F(0,-\mathfrak{w})} \\ &= -8\widetilde{N}(\pi T)^2 \frac{F'(0,\mathfrak{w})}{F(0,\mathfrak{w})} \,, \end{aligned} \tag{C.9}$$

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and the conductivity is

$$\begin{aligned}\sigma_q &= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}G^{\mathrm{ret}}_{xx}(K)\Big|_{\omega=|\vec{k}|}\\ &= 8\widetilde{N}(\pi T)^2\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\left(\frac{F_1'(0,\mathfrak{w})\mathfrak{w}}{F_0}+\mathcal{O}(\omega^2)\right)\\ &= \frac{N_c N_f T}{4\pi}\ .\end{aligned}\tag{C.10}$$

Extending the procedure to finite density yields [26]

$$\sigma_q = \frac{N_c N_f T}{4\pi}\sqrt{1+\mathbf{d}^2}\,,\tag{C.11}$$

where $\mathbf{d} = \frac{d}{(\pi T)^3} = \frac{8n_q}{N_c N_f\sqrt{\lambda}T^3}$, which is (B.4) with $c=1$.

D. D4/D8 deconfined phase

In this section we enforce an eigenmode analysis on the longitudinal and transverse vector currents in the deconfined hot and dense D4/D8. The eigenmode analysis parallels the one for cold and dense D4/D8 with reflective boundary conditions at the BH horizon. No imaginary parts arise from this analysis. In a way, in D4/D8 we may still entertain the possibility of stationary solutions between the Left-pending and Right-pending branes to mock up existing light bound states. Of course, this is a formal suggestion.

The longitudinal operator ($\mathcal{D}_L$) is

$$\mathcal{D}_L \equiv \partial_Z\frac{\sqrt{K(K-1)}\Delta^3}{\Delta^2\omega^2-\frac{K-1}{K}q^2}\partial_Z+\frac{K^{1/6}}{\sqrt{K-1}}\Delta\ .\tag{D.1}$$

When $q=0$ or $\omega=0$ it is easily diagonalized, since

$$\begin{aligned}\mathcal{D}_L(q=0) &= \frac{1}{\omega^2}\partial_Z\sqrt{K(K-1)}\Delta\partial_Z+\frac{K^{1/6}}{\sqrt{K-1}}\Delta\,,\\ \mathcal{D}_L(\omega=0) &= -\frac{1}{q^2}\partial_Z\frac{K^{3/2}}{\sqrt{K-1}}\Delta^3\partial_Z+\frac{K^{1/6}}{\sqrt{K-1}}\Delta\ .\end{aligned}\tag{D.2}$$

The Green's function ($\mathcal{D}_L^{-1}$) may be expanded in terms of the complete set of eigenvalues that diagonalize

$$\mathcal{D}_L(q=0)f = \left(\frac{K^{1/6}}{\omega^2\sqrt{K-1}}\Delta\right)\lambda f\,,\tag{D.3}$$

$$\mathcal{D}_L(\omega=0)f = \left(\frac{K^{1/6}}{q^2\sqrt{K-1}}\Delta\right)\lambda f\,,\tag{D.4}$$

where $\frac{K^{1/6}}{\omega^2\sqrt{K-1}}\Delta$ and $\frac{K^{1/6}}{q^2\sqrt{K-1}}\Delta$ are weight factors. Using the complete sets,

$$\begin{aligned}\left(\partial_Z\sqrt{K(K-1)}\Delta\partial_Z\right)\chi_n &= -\left(\frac{K^{1/6}}{\omega^2\sqrt{K-1}}\Delta\right)(\lambda_n^\chi)^2\,\chi_n\,,\\ \left(\partial_Z\frac{K^{3/2}}{\sqrt{K-1}}\Delta^3\partial_Z\right)\xi_n &= -\left(\frac{K^{1/6}}{\omega^2\sqrt{K-1}}\Delta\right)(\lambda_n^\xi)^2\,\xi_n\,,\end{aligned}$$

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we have

$$\langle Z|\mathcal{D}_L^{-1}(q=0)|Z'\rangle = \sum_{n\in\mathbb{N}} \frac{\chi_n(Z)\chi_n(Z')}{-\omega^2+(\lambda_n^\chi)^2} + \frac{\chi_0(Z)\chi_0(Z')}{\omega^2}\,, \tag{D.5}$$

$$\langle Z|\mathcal{D}_L^{-1}(\omega=0)|Z'\rangle = \sum_{n\in\mathbb{N}} \frac{\xi_n(Z)\xi_n(Z')}{q^2+(\lambda_n^\xi)^2} + \frac{\xi_0(Z)\xi_0(Z')}{q^2}\,. \tag{D.6}$$

The transversal operator ($\mathcal{D}_T$) is

$$\mathcal{D}_T \equiv -\frac{1}{\omega^2}\left(\partial_Z\sqrt{K(K-1)}\Delta\partial_Z + \frac{K^{1/6}}{\sqrt{K-1}}\left(\Delta\omega^2 - \Delta^{-1}\frac{K-1}{K}q^2\right)\right)\,. \tag{D.7}$$

When $q=0$ it is easily diagonalized as

$$\mathcal{D}_T(q=0) \equiv -\frac{1}{\omega^2}\partial_Z\sqrt{K(K-1)}\Delta\partial_Z - \frac{K^{1/6}}{\sqrt{K-1}}\Delta\,. \tag{D.8}$$

With the eigenfunctions and eigenvalues:

$$(\partial_Z\sqrt{K(K-1)}\Delta\partial_Z)\,\zeta_n = -\frac{K^{1/6}}{\sqrt{K-1}}\Delta\,(\lambda_n^\zeta)^2\,\zeta_n\,,$$

the Green's function is expanded as

$$\langle Z|\mathcal{D}_T^{-1}(q=0)|Z'\rangle = \sum_{n\in\mathbb{N}} \frac{\zeta_n(Z)\zeta_n(Z')}{\omega^2+(\lambda_n^\zeta)^2} + \frac{\zeta_0(Z)\zeta_0(Z')}{\omega^2}\,. \tag{D.9}$$

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